BU
AF-4561/13

Division of Engineering
BROWN UNIVERSITY
PROVIDENCE, R. I.

AFCRL 949

EQUIVALENT CIRCUIT PARAMETER FOR AN INHOMOGENEOUS BIFURCATED WAVEGUIDE

BY

H. M. CRONSON

Air Force Cambridge Research Laboratories
Office of Aerospace Research
Contract AF 19(604)-4561 —
Scientific Report AF-4561/13

September 1961

AFCRL 949

EQUIVALENT CIRCUIT PARAMETER FOR AN INHOMOGENEOUS BIFURCATED WAVEGUIDE

By

H. M. Cronson

Scientific Report AF 4561/13

DIVISION OF ENGINEERING

BROWN UNIVERSITY

PROVIDENCE, RHODE ISLAND

September 1961

Contract Monitor: Dr. Werner W. Gerbes

"The research reported in this document has been sponsored in part by the Electronics Research Directorate of the Air Force Cambridge Research Laboratories, Office of Aerospace Research, and by the Office of Naval Research and the David Taylor Model Basin. The publication of this report does not necessarily constitute approval by the Air Force of the findings or conclusions contained herein."

Contract title: Research Directed toward the study of Radiation of Electromagnetic Waves

Contract number: AF 19(604)-4561

"Requests for additional copies by Agencies of the Department of Defense, their contractors, and other Government agencies should be directed to the:

ARMED SERVICES TECHNICAL INFORMATION AGENCY
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA"

"All other persons and organizations should apply to the:

U. S. DEPARTMENT OF COMMERCE
OFFICE OF TECHNICAL SERVICES
WASHINGTON 25, D.C."

TABLE OF CONTENTS

ABSTRACT

In this report an approximate solution for an equivalent circuit representation of a two dimensional waveguide junction is obtained over several ranges of significant parameters. The junction is formed by a semi-infinite, dielectric filled, parallel plate guide contained symmetrically within an infinite parallel plate guide. The structure is first reduced to a half-space problem and formulated in terms of the appropriate field modes in each of the parallel plate guides forming the junction. These modes are matched at the plane of discontinuity and the resulting equations rearranged to resemble ordinary circuit equations. The circuit elements of the equivalent circuit representation are identified from the equations. The determination of the values of these elements requires the solution of an infinite set of inhomogeneous algebraic equations. Solutions are obtained by approximate methods with the aid of an IBM 7070 computer. The approximate solutions are checked by comparing the results obtained for the degenerate case, in which the dielectric vanishes, with known exact results. This check suggests that the approximate results for the case of non-vanishing dielectric should be within a few per cent of the correct values.

I. Introduction

This report concerns the determination of an equivalent circuit representation for a waveguide junction formed by a semi-infinite, dielectric filled, parallel plate guide contained symmetrically within an infinite parallel plate guide (see Fig. 1). We will assume that the dimensions of the guide are such that only the T.E.M. mode will propagate in each guide at the excitation frequency, and that the exciting source is so polarized that there will be no field variations in the x-direction.

The literature on waveguide discontinuities discloses the closely related problem of a homogeneous E-plane bifurcation, which can be solved exactly using integral transform methods.[1] However a preliminary investigation of these transform procedures applied to the problem considered in this report revealed that the inhomogeneity introduced by the dielectric in guide B made these methods impractical to apply. Another approach, better suited to the type of discontinuity considered here, is an approximate method originally due to Whinnery and Jamieson.[2] Their approach is based on mode matching at the discontinuity. It has the advantage of being relatively straightforward, inherently providing a clear physical picture of the propagating and non-propagating waves in the guide. Moreover, it is particularly adaptable to the two dimensional equivalent

---

[1] N. Marcuvitz, "Waveguide Handbook", M. I. T. Radiation Laboratory Series, Vol. 10 (McGraw-Hill Book Company, Inc., 1951), pp. 160-167.

[2] J. R. Whinnery and H. W. Jamieson, "Equivalent Circuits for Discontinuities in Transmission Lines", Proc. I.R.E., 32, 98-114, Feb., 1944.

circuit formulation of the problem.

In the structure examined here, the method consists of finding the evanescent mode amplitudes excited in the guide of height 2a. After considerable manipulation of the field equations at the plane $Z = 0$, we obtain an expression for the equivalent capacitances of the junction in terms of these non-propagating mode amplitudes which, in turn, are defined by an infinite set of non-homogeneous equations. An approximate solution to these equations over a convenient range of parameters is obtained by machine calculation using an IBM 7070 computer. The procedure is checked by comparing the results for the degenerate case, in which $\epsilon_B = \epsilon_0$, with the values given by Marcuvitz.[1]

II. Field Equations

A. Reduction to a Half-Space Problem.

The given waveguide configuration can be simplified by the following considerations.

(1) Due to the method of excitation and the nature of the discontinuity, there will be no H-modes in the guide.

(2) The symmetry of the structure allows us to place an infinite perfectly conducting plane at the plane $y = 0$ without disturbing the fields.

Thus instead of solving the original problem, it is sufficient to solve the half-space problem shown in Fig. 2. (See Appendix of reference 3 for the justification of these statements.)

B. Field Representation in the Half-Space Guide.

The field in each of the guides shown in Fig. 2 can be represented by an incident and reflected T.E.M. mode plus an infinite sum of evanescent E modes. Using harmonic time dependence, e^{jwt}, and assuming T.E.M. sources at frequency in guides B and C at $Z = -\infty$ and in guide A at $Z = \infty$, the y component of the electric field in guide B is given by:

$$E_{yB}(y,z) = B_0^i e^{-jk_B z} + B_0^r e^{jk_B z} + \sum_{n=1}^{\infty} B_n e^{\gamma_n^B z} \cos \frac{n\pi}{b} y \tag{1}$$

for $0 \leq y < b$ and $z \leq 0$

where $k_B = \omega\sqrt{\mu_0 \epsilon_B}$ and $\gamma_n^B = \sqrt{\left(\frac{n\pi}{b}\right)^2 - k_B^2}$ are always real.

[3] H. M. Cronson, "Equivalent Circuit Parameters for an Inhomogeneous Bifurcated Waveguide", Master's thesis, Brown University, (1961).

The x component of magnetic field in guide B can be obtained from Maxwell's Equation $\nabla \times \underline{H}_B = j\omega\epsilon_B \underline{E}_B$, using the fact that since only E modes are present, $H_y = H_z = 0$. This yields

$$H_{xB}(y,z) = -\sqrt{\frac{\epsilon_B}{\mu_0}}\left[B_0^i e^{-jk_B z} - B_0^r e^{jk_B z}\right] + j\omega\epsilon_B \sum_{m=1}^{\infty} \frac{B_m}{\gamma_m^B} e^{\gamma_m^B z} \cos\frac{m\pi}{b}y \qquad (2)$$

for $0 \leq y < b$ and $z \leq 0$

Similarly, for the fields in Guide A we can write

$$E_{yA}(y,z) = A_0^i e^{jkz} + A_0^r e^{-jkz} + \sum_{m=1}^{\infty} A_m e^{-\gamma_m^A z} \cos\frac{m\pi}{a}y \qquad (3)$$

$$H_{xA}(y,z) = \sqrt{\frac{\epsilon_0}{\mu_0}}\left[A_0^i e^{jkz} - A_0^r e^{-jkz}\right] - j\omega\epsilon_0 \sum_{m=1}^{\infty} \frac{A_m}{\gamma_m^A} e^{-\gamma_m^A z} \cos\frac{m\pi}{a}y \qquad (4)$$

for $0 \leq y \leq a$ and $z > 0$

where $k = \omega\sqrt{\mu_0\epsilon_0}$ and $\gamma_m^A = \sqrt{\left(\frac{m\pi}{a}\right)^2 - k^2}$

Also, in Guide C, letting $y' = y-b$,

$$E_{yC}(y',z) = C_0^i e^{-jkz} + C_0^r e^{jkz} + \sum_{q=1}^{\infty} C_q e^{\gamma_q^C z} \cos\frac{q\pi}{c}y' \qquad (5)$$

$$H_{xC}(y',z) = -\sqrt{\frac{\epsilon_0}{\mu_0}}\left[C_0^i e^{-jkz} - C_0^r e^{jkz}\right] + j\omega\epsilon_0 \sum_{q=1}^{\infty} \frac{C_q}{\gamma_q^C} e^{\gamma_q^C z} \cos\frac{q\pi}{c}y' \qquad (6)$$

for $0 < y' \leq C$ and $z \leq 0$

where $\gamma_g^c = \sqrt{\left(\frac{g\pi}{c}\right)^2 - k^2}$

For subsequent calculations it is convenient to define the following quantities,

$$
\begin{aligned}
A_0 &= A_0^i + A_0^r & \qquad A_0 Y_{A_0} &= \sqrt{\frac{\epsilon_0}{\mu_0}}\left[A_0^i - A_0^r\right] \\
B_0 &= B_0^i + B_0^r & \qquad B_0 Y_{B_0} &= -\sqrt{\frac{\epsilon_0}{\mu_0}}\left[B_0^i - B_0^r\right] \\
C_0 &= C_0^i + C_0^r & \qquad C_0 Y_{C_0} &= -\sqrt{\frac{\epsilon_0}{\mu_0}}\left[C_0^i - C_0^r\right]
\end{aligned}
\tag{6a}
$$

$$Y_{A_m} = -\frac{j\omega\epsilon_0}{\gamma_m^A} \qquad m = 1, 2, 3, \cdots \tag{7}$$

$$Y_{B_n} = -\frac{j\omega\epsilon_B}{\gamma_n^B} \qquad n = 1, 2, 3, \cdots \tag{8}$$

$$Y_{C_g} = \frac{j\omega\epsilon_0}{\gamma_g^c} \qquad g = 1, 2, 3, \cdots \tag{9}$$

Hence equations (1) through (6) can be written, at $Z = 0$, as

$$E_{yB}(y, 0) = B_0 + \sum_{n=1}^{\infty} B_n \cos\frac{n\pi}{b} y \tag{10}$$

$$H_{xB}(y, 0) = B_0 Y_{B_0} + \sum_{n=1}^{\infty} Y_{B_n} B_n \cos\frac{n\pi}{b} y \tag{11}$$

$$E_{yA}(y,0) = A_0 + \sum_{m=1}^{\infty} A_m \cos\frac{m\pi}{a}y \tag{12}$$

$$H_{xA}(y,0) = A_0 Y_{A_0} + \sum_{m=1}^{\infty} Y_{A_m} A_m \cos\frac{m\pi}{a}y \tag{13}$$

$$E_{yC}(y',0) = C_0 + \sum_{q=1}^{\infty} C_q \cos\frac{q\pi}{c}y' \tag{14}$$

$$H_{xC}(y',0) = C_0 Y_{C_0} + \sum_{q=1}^{\infty} Y_{C_q} C_q \cos\frac{q\pi}{c}y' \tag{15}$$

We now employ the condition of continuity of the tangential E and H fields at Z = 0 to give,

$$B_0 + \sum_{n=1}^{\infty} B_n \cos\frac{n\pi}{b}y = A_0 + \sum_{m=1}^{\infty} A_m \cos\frac{m\pi}{a}y \qquad 0 \le y < b \tag{16}$$

$$C_0 + \sum_{q=1}^{\infty} C_n \cos\frac{n\pi}{c}y' = A_0 + \sum_{m=1}^{\infty} A_m \cos\frac{m\pi}{a}y \qquad \begin{matrix} b < y \le a \\ 0 < y' \le c \end{matrix} \tag{17}$$

$$Y_{B_0} B_0 + \sum_{n=1}^{\infty} Y_{B_n} B_n \cos\frac{n\pi}{b}y = Y_{A_0} A_0 + \sum_{m=1}^{\infty} Y_{A_m} A_m \cos\frac{m\pi}{a}y \qquad 0 \le y < b \tag{18}$$

$$Y_{C_0} C_0 + \sum_{q=1}^{\infty} Y_{C_q} C_n \cos\frac{q\pi}{c}y' = Y_{A_0} A_0 + \sum_{m=1}^{\infty} Y_{A_m} A_m \cos\frac{m\pi}{a}y \qquad \begin{matrix} b < y \le a \\ 0 < y' \le c \end{matrix} \tag{19}$$

The above relationships between the A_m, B_n, and C_q, will be utilized later to provide an equivalent circuit representation for the waveguide junction.

III. Equivalent Circuit Representation

A. The Transmission Line Analogy of a Parallel Plate Waveguide.

Consider an infinite parallel plate waveguide of height d in which the T.E.M. mode is propagating in the positive Z direction (see Fig. 3). The T.E.M. wave may be written for harmonic time dependence e^{jwt} as

$$E_y = De^{-jkz} \qquad H_x = -\sqrt{\frac{\epsilon}{\mu}}\, De^{-jkz}$$

where D is a constant determined by the strength of the source.

Maxwell's equations within the waveguide may be written in the form

$$\nabla \times \underline{E} = -j\omega\mu H \qquad \nabla \times \underline{H} = j\omega\epsilon \underline{E}$$

or as

$$\frac{\partial E_y}{\partial z} = j\omega\mu H_x \tag{20}$$

$$\frac{\partial H_x}{\partial z} = j\omega\epsilon E_y \tag{21}$$

We want to define a "voltage" depending on Ey and a "current" depending on Hx so that equations (20) and (21) can be viewed as transmission line equations. We define

$$V(z) = -\int_0^d E_y\, dy \tag{22}$$

For the parallel plate guide

$$V(z) = -E_y d \tag{23}$$

We also define

$$I(z,d) = w\underline{\ell}\cdot\hat{z}_0 \tag{24}$$

where $\underline{\ell}$ is the current per unit length on the surface of the top plate ($y = d$), w is a length in the x direction, and $\hat{z}_0$ is the unit vector in the Z direction, (see Fig. 3). On the surface of a perfect conductor $\hat{\underline{n}} \times \underline{H} = \underline{\ell}$ where for this case $\hat{n}$ is the normal pointing into the guide.

Similarly

$$I(z,0) = w\underline{\ell}\cdot\hat{z}_0$$

We note

$$\underline{\ell}\Big|_{y=d} = (-\hat{y}_0)\times H_x\hat{x}_0 = H_x\hat{z}_0$$

$$\underline{\ell}\Big|_{y=0} = (\hat{y}_0)\times H_x\hat{x}_0 = -H_x\hat{z}_0$$

Therefore

$$I(z,d) = wH_x$$

$$I(z,0) = -wH_x$$

We further define

$$I(z) = I(z,d) = wH_x = -I(z,0) \tag{25}$$

Substitution of equations (23) and (25) into (20) and (21) yields

$$\frac{dV(z)}{dz} = -j\omega\mu\frac{d}{w}I(z) \tag{26}$$

$$\frac{dI(z)}{dz} = -j\omega\epsilon \frac{w}{d} V(z) \qquad (27)$$

The appropriate transmission line equations are given by Marcuvitz (op. cit.) in the form

$$\frac{d\mathcal{V}}{dz} = -jkZ\mathcal{I} \qquad (28)$$

$$\frac{d\mathcal{I}}{dz} = -jkY\mathcal{V} \qquad (29)$$

where $\mathcal{V}$ and $\mathcal{I}$ are the voltage and current respectively in the transmission line, $k = \omega\sqrt{\mu\epsilon}$, and $Z = 1/Y$ is the characteristic impedance. In comparing equations (26) and (27) with (28) and (30), we see that the T.E.M. mode in a parallel plate waveguide can be completely represented by a transmission line with characteristic impedance.

$$Z = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}} = Z_d^0 \qquad (29\text{-a})$$

Note that the power flow in the guide through a cross section w x d is

$$P = \int_0^d\int_0^w \frac{1}{2} Re\left[\underline{E}\times\underline{H}^*\right]\cdot\hat{z}_0\, dx\, dy = -\frac{wd}{2} E_y H_x^* = \frac{1}{2} V(z) I^*(z)$$

which is the expression for the power flow in a transmission line.

B. The Waveguide Junction as an Equivalent Circuit.

Since a parallel plate waveguide can be represented by a transmission line, it is convenient to represent the junction by a lumped network. The junction in Fig. 2 stores energy with no dissipation in a local region close to the plane $Z = 0$. Thus it is reasonable to represent the junction by a lossless, lumped network. In this section we will show that the field equations at the junction, expressed in terms of our defined voltage and current, correspond to the nodal equations for a certain lumped, lossless, and passive network.

The voltage from 0 to b in guide B at the plane $Z = 0$ is determined using equations (10) and (22):

$$V(0)\Big|_{\text{guide } B} = V_{0B} = -\int_0^b E_{yB}(y,0) = -\int_0^b \left[B_0 + \sum_{n=1}^{\infty} B_n \cos\frac{n\pi}{b}y \right] dy = -B_0 b$$

The voltage from 0 to a in guide A and from 0 to c in guide C at $Z = 0$ are found to be

$$V_{0A} = -A_0 a \qquad\qquad V_{0C} = -C_0 c \qquad (29\text{-b})$$

We note that, since the cosine dependent modes integrate to 0, the voltages at $Z = 0$ are due only to the electric field of the dominant T.E.M. mode.

If equation (16) is integrated with respect to y from 0 to b and added to the result obtained when equation (17) is integrated with respect to y from b to c, one obtains

$$B_0 b + C_0 c = A_0 a$$

or in terms of voltages,

$$V_{0B} + V_{0C} = V_{0A} \qquad (30)$$

The defined current on the top plate in guide B at Z = 0 can be found from equations (25) and (11) as

$$I(0)\Big|_{\text{guide }B} = I_B = \omega H_{xB}(b,0) = \omega\left[B_0 Y_{B0} + \sum_{n=1}^{\infty} Y_{Bn} B_n (-1)^n\right]$$

Similarly, the defined current on the top plate in guide C and guide A at Z = 0 can be written as

$$I_c = \omega\left[C_0 Y_{c0} + \sum_{q=1}^{\infty} Y_{cq} C_q (-1)^q\right]$$

$$I_A = \omega\left[A_0 Y_{A0} + \sum_{m=1}^{\infty} Y_{Am} A_m (-1)^m\right]$$

We designate the currents of the dominant mode as

$$I_{0A} = \omega A_0 Y_{A0} \qquad I_{0B} = \omega B_0 Y_{B0} \qquad I_{0C} = \omega C_0 Y_{C0}$$

Integrating equation (18) with respect to y from 0 to b we obtain

$$Y_{B0} B_0 = Y_{A0} A_0 + \frac{a}{\pi b}\sum_{m=1}^{\infty} \frac{Y_{Am} A_m}{m} \sin m\pi \frac{b}{a}$$

Substituting equation (7) we have

$$Y_{B0} B_0 = Y_{A0} A_0 + \frac{a}{\pi b}\sum_{m=1}^{\infty}\left[-\frac{j\omega\epsilon_0}{\gamma_m^A}\right]\frac{A_m}{m} \sin m\pi \frac{b}{a} \tag{31-a}$$

We can also show that equation (19) can be written

$$Y_{C0} C_0 = Y_{A0} A_0 - \frac{a}{\pi c}\sum_{m=1}^{\infty}\left[-\frac{j\omega\epsilon_0}{\gamma_m^A}\right]\frac{A_m}{m} \sin m\pi \frac{b}{a} \tag{32-a}$$

Suppose the summation term in equations (31-a) and (32-a) is multiplied by $\dfrac{B_0 - C_0}{B_0 - C_0}$ and we define

$$F = \frac{a\epsilon_0}{\pi} \sum_{m=1}^{\infty} \frac{A_m \sin m\pi \frac{b}{a}}{[B_0 - C_0]\, \gamma_m^A\, m} \tag{33}$$

We may then write equations (31-a) and (32-a) as

$$Y_{B_0} B_0 = Y_{A_0} A_0 - j\omega \frac{F}{b} B_0 + j\omega \frac{F}{b} C_0$$

$$Y_{C_0} C_0 = Y_{A_0} A_0 + j\omega \frac{F}{c} B_0 - j\omega \frac{F}{c} C_0$$

Using these equations and the definitions of the currents in terms of the admittances, we obtain

$$I_{0B} = I_{0A} + j\omega \frac{F}{b^2} w V_{0B} - j\omega \frac{F}{bc} w V_{0C} \tag{31-b}$$

$$I_{0C} = I_{0A} - j\omega \frac{F}{bc} w V_{0B} + j\omega \frac{F}{c^2} w V_{0C} \tag{32-b}$$

Substituting $V_{OC} = V_{OA} - V_{OB}$ and $V_{OB} = V_{OA} - V_{OC}$ in equations (31-b) and (32-b) and remembering that $a = b + c$, we have

$$I_{0B} = I_{0A} + j\omega \frac{aF}{b^2 c} w V_{0B} - j\omega \frac{F}{bc} w V_{0A}$$

$$I_{0C} = I_{0A} + j\omega \frac{aF}{bc^2} w V_{0C} - j\omega \frac{F}{bc} w V_{0A}$$

Define

$$\frac{aF\omega}{b^2c} = C_B \qquad (34)$$

$$\frac{aF\omega}{bc^2} = C_C \qquad (35)$$

$$-\frac{F\omega}{bc} = C_A \qquad (36)$$

Substituting these definitions in the previous equations yields

$$I_{0B} = I_{0A} + j\omega C_B V_{0B} + j\omega C_A V_{0A}$$

$$I_{0C} = I_{0A} + j\omega C_C V_{0C} + j\omega C_A V_{0A}$$

which are the nodal equations of the network shown in Fig. 4. Thus, the waveguide junction can be represented by a lumped, lossless, and passive network.

IV. Determination of the Network Capacitances

From equations (34), (35), (36), we see that the capacitances could be determined if F were known. To find F we must find $\frac{A_m}{B_o - C_o}$. We shall now determine equations for this unknown.

We multiply equation (16) by $\cos \frac{m' \pi y}{a}$ and integrate with respect to y from 0 to b. This equation is then added to the result obtained when equation (17) is multiplied by $\cos \frac{m' \pi y}{a}$ and integrated with respect to y from b to c. These operations result in

$$A_m = \frac{2 \sin m\pi \frac{b}{a}}{m\pi} \left[B_o - C_o\right] + \frac{2m}{\pi}\left(\frac{b}{a}\right)^2 \sin m\pi \frac{b}{a} \sum_{\substack{n=1 \\ n \neq \frac{b}{a} m}}^{\infty} \frac{(-1)^n B_n}{n^2\left\{\left(\frac{mb}{na}\right)^2 - 1\right\}} + \frac{b}{a}\sum_{n=1}^{\infty} \delta_{n, \frac{b}{a} m} B_n$$

$$- \frac{2m}{\pi}\left(\frac{c}{a}\right)^2 \sin m\pi \frac{b}{a} \sum_{\substack{q=1 \\ q \neq \frac{c}{a} m}}^{\infty} \frac{C_q}{q^2\left\{\left(\frac{mc}{qa}\right)^2 - 1\right\}} + \frac{c}{a}(-1)^{\frac{mb}{a}} \sum_{q=1}^{\infty} \delta_{q, \frac{c}{a} m} C_q \qquad (37)$$

where

$$\delta_{n, \frac{b}{a} m} = \begin{cases} 0 & \text{if } n \neq \frac{b}{a} m \\ 1 & \text{if } n = \frac{b}{a} m \end{cases}$$

Next we multiply equation (18) by $\cos \frac{m' \pi y}{b}$ and integrate with respect to y from 0 to b. The result is

$$Y_{B_n} B_n = \frac{2b(-1)^n}{n^2 a \pi} \sum_{\substack{m=1 \\ m \neq n\frac{a}{b}}}^{\infty} \frac{m Y_{A_m} A_m \sin m\pi \frac{b}{a}}{\left\{\left(\frac{mb}{na}\right)^2 - 1\right\}} + \sum_{m=1}^{\infty} \delta_{m, \frac{na}{b}} Y_{A_m} A_m \qquad (38)$$

Equation (19) is multiplied by $\cos \frac{q' \pi y'}{c}$ and integrated with respect to y' from 0 to C. This gives

$$Y_{c_q} C_q = -\frac{2c}{q^2 a\pi}\sum_{\substack{m=1\\ m\neq q\frac{a}{c}}}^{\infty} \frac{m Y_{A_m} A_m \sin m\pi\frac{b}{a}}{\left\{\left(\frac{mc}{qa}\right)^2 - 1\right\}} + \sum_{m=1}^{\infty} \delta_{m,\frac{aq}{c}} (-1)^{\frac{mb}{a}} Y_{A_m} A_m \tag{39}$$

Substituting the B_m and C_q from equations (38) and (39) into (37) yields

$$\overset{(1)}{A_m} = \overset{(2)}{\frac{2\sin m\pi b/a}{m\pi}\left[B_o - C_o\right]} - \overset{(3)}{\frac{4mb^3\sin m\pi b/a}{\varepsilon_B\pi^2 a^3}\sum_{\substack{n=1\\ n\neq\frac{b}{a}m}} \frac{\gamma_n^B}{n^4\left\{\left(\frac{mb}{na}\right)^2-1\right\}}\sum_{\substack{p=1\\ p\neq\frac{a}{b}m}} \frac{pA_p\sin p\pi\frac{b}{a}}{\gamma_p^A\left\{\left(\frac{pb}{na}\right)^2-1\right\}}}$$

$$\overset{(4)}{-\frac{2mb^2\sin m\pi b/a}{\varepsilon_B\pi a^2}\sum_{\substack{n=1\\ n\neq\frac{b}{a}m}}^{\infty} \frac{(-1)^n\gamma_n^B}{n^2\left\{\left(\frac{mb}{na}\right)^2-1\right\}}\sum_{p=1}^{\infty}\frac{\delta_{p,n\frac{a}{b}}A_p}{\gamma_p^A}}$$

$$\overset{(5)}{-\frac{2b^2}{\varepsilon_B a^2\pi}\sum_{n=1}^{\infty}\frac{\delta_{n,\frac{b}{a}m}(-1)^n\gamma_n^B}{n^2}\sum_{\substack{p=1\\ p\neq\frac{a}{b}m}}^{\infty}\frac{pA_p\sin p\pi\frac{b}{a}}{\gamma_p^A\left\{\left(\frac{pb}{na}\right)^2-1\right\}}}$$

$$\overset{(6)}{-\frac{b}{\varepsilon_B a}\sum_{n=1}^{\infty}\delta_{n,\frac{b}{a}m}\gamma_n^B\sum_{p=1}^{\infty}\frac{\delta_{p,n\frac{a}{b}}A_p}{\gamma_p^A}}$$

$$\overset{(7)}{-\frac{4mc^3\sin m\pi\frac{b}{a}}{\pi^2 a^3}\sum_{\substack{q=1\\ q\neq\frac{c}{a}m}}^{\infty}\frac{\gamma_q^C}{q^4\left\{\left(\frac{mc}{qa}\right)^2-1\right\}}\sum_{\substack{p=1\\ p\neq q\frac{a}{c}}}^{\infty}\frac{pA_p\sin p\pi\frac{b}{a}}{\gamma_p^A\left\{\left(\frac{pc}{qa}\right)^2-1\right\}}} \quad +$$

(equation continued on next page)

8

$$+\frac{2mc^2\sin m\pi b/a}{\pi^2 a^2}\sum_{\substack{q=1\\ q\neq \frac{c}{a}m}}^{\infty}\frac{\gamma_q^c}{q^2\left\{\left(\frac{mc}{qa}\right)^2-1\right\}}\sum_{p=1}^{\infty}\frac{\delta_{p,\frac{aq}{c}}(-1)^{p\frac{b}{a}}A_p}{\gamma_p^A}$$

9

$$+\frac{2c^2(-1)^{m\frac{b}{a}}}{\pi a^2}\sum_{q=1}^{\infty}\frac{\delta_{q,\frac{c}{a}m}\gamma_q^c}{q^2}\sum_{\substack{p=1\\ p\neq q\frac{a}{c}}}\frac{pA_p\sin p\pi b/a}{\gamma_p^A\left\{\left(\frac{pc}{qa}\right)^2-1\right\}}$$

10

$$-\frac{c(-1)^{m\frac{b}{a}}}{a}\sum_{q=1}^{\infty}\delta_{q,\frac{c}{a}m}\gamma_q^c\sum_{p=1}^{\infty}\frac{\delta_{p,\frac{aq}{c}}(-1)^{p\frac{b}{a}}A_p}{\gamma_p^A} \tag{40}$$

where $\mathcal{E}_B = \dfrac{\epsilon_B}{\epsilon_0}$ and the encircled numbers designate the terms of the equation for subsequent identification.

Next, each term is rearranged so that Ap appears only under the first summation. For instance term 3 is rearranged by noting that

$$\gamma_n^B = \frac{\pi}{b}\left[n^2 - \left(\frac{2b}{\lambda_b}\right)^2\right]^{\frac{1}{2}}$$

where λ_b is the wavelength in guide B and also by changing the order of summation

Term 3 becomes

$$-\frac{4m}{\mathcal{E}_B\,\pi}\left(\frac{b}{a}\right)^2 \sin m\pi\frac{b}{a}\sum_{p=1}^{\infty}\frac{p A_p \sin p\pi\frac{b}{a}}{a\;\gamma_p^A}\sum_{\substack{n=1\\ n\neq\frac{b}{a}m,\,\frac{b}{a}p}}\frac{\left[n^2-\left(\frac{2b}{\lambda_b}\right)^2\right]^{\frac{1}{2}}}{n^4\left\{\left(\frac{mb}{na}\right)^2-1\right\}\left\{\left(\frac{pb}{na}\right)^2-1\right\}}$$

In order to group all the Am terms together in equation (40) we also separate term 3 into two parts. One including all the Ap save Am and the other just containing Am. When all the terms have been suitably rearranged equation (40) becomes

$$\overset{2}{\frac{2\sin\pi m\frac{b}{a}}{m\pi}} = X(m)\left\{\overset{1}{\pi\left[m^2-\left(\frac{2a}{\lambda_a}\right)^2\right]^{\frac{1}{2}}} + \pi\,\delta\!\left(\tfrac{b}{a}m,\text{int}\right)\left[\overset{6}{\frac{1}{\mathcal{E}_B}\left[\left(\frac{bm}{a}\right)^2-\left(\frac{2b}{\lambda_b}\right)^2\right]^{\frac{1}{2}}} + \overset{10}{\left[\left(\frac{cm}{a}\right)^2-\left(\frac{2c}{\lambda_a}\right)^2\right]^{\frac{1}{2}}}\right]\right.$$

$$\left. + \frac{4m^2\sin^2\!\left(m\pi\frac{b}{a}\right)}{\pi}\left[\overset{3b}{\frac{1}{\mathcal{E}_B}\left(\frac{b}{a}\right)^3\sum_{\substack{n=1\\ n\neq\frac{b}{a}m}}^{\infty}\frac{\left[n^2-\left(\frac{2b}{\lambda_b}\right)^2\right]^{\frac{1}{2}}}{n^4\left\{\left(\frac{mb}{na}\right)^2-1\right\}^2}} + \overset{7b}{\left(\frac{c}{a}\right)^2\sum_{\substack{n=1\\ n\neq\frac{c}{a}m}}^{\infty}\frac{\left[n^2-\left(\frac{2c}{\lambda_a}\right)^2\right]^{\frac{1}{2}}}{n^4\left\{\left(\frac{mc}{na}\right)^2-1\right\}^2}}\right]\right\} +$$

(equation continued on next page)

$$+\sum_{\substack{p=1\\p\neq m}}^{\infty} X(p)\left\{\frac{2m\sin\left(m\pi\frac{b}{a}\right)\delta\left(p\frac{b}{a},int\right)(-1)^{p\frac{b}{a}}}{m^2-p^2}\left[\overset{4}{\frac{1}{\epsilon_B\left(\frac{b}{a}\right)}\left[\left(p\frac{b}{a}\right)^2-\left(\frac{2b}{\lambda_b}\right)^2\right]^{\frac{1}{2}}}-\overset{8}{\frac{1}{\left(\frac{c}{a}\right)}\left[\left(p\frac{c}{a}\right)^2-\left(\frac{2c}{\lambda_a}\right)^2\right]^{\frac{1}{2}}}\right]\right.$$

$$+\frac{2p\sin\left(p\pi\frac{b}{a}\right)\delta\left(m\frac{b}{a},int\right)(-1)^{m\frac{b}{a}}}{p^2-m^2}\left[\overset{5}{\frac{1}{\epsilon_B\left(\frac{b}{a}\right)}\left[\left(\frac{mb}{a}\right)^2-\left(\frac{2b}{\lambda_b}\right)^2\right]^{\frac{1}{2}}}-\overset{9}{\frac{1}{\left(\frac{c}{a}\right)}\left[\left(\frac{mc}{a}\right)^2-\left(\frac{2c}{\lambda_a}\right)^2\right]^{\frac{1}{2}}}\right]$$

$$\left.+\frac{4mp\sin\left(m\pi\frac{b}{a}\right)\sin\left(p\pi\frac{b}{a}\right)}{\pi}\left[\overset{3a}{\frac{1}{\epsilon_B}\left(\frac{b}{a}\right)^2\sum_{\substack{n=1\\n\neq\frac{b}{a}m,\frac{b}{a}p}}^{\infty}\frac{\left[n^2-\left(\frac{2b}{\lambda_b}\right)^2\right]^{\frac{1}{2}}}{n^4\left\{\left(\frac{mb}{na}\right)^2-1\right\}\left\{\left(\frac{pb}{na}\right)^2-1\right\}}}+\overset{7b}{\left(\frac{c}{a}\right)^2\sum_{\substack{n=1\\n\neq\frac{c}{a}m,\frac{c}{a}p}}^{\infty}\frac{\left[n^2-\left(\frac{2c}{\lambda_a}\right)^2\right]^{\frac{1}{2}}}{n^4\left\{\left(\frac{mc}{na}\right)^2-1\right\}\left\{\left(\frac{pc}{na}\right)^2-1\right\}}}\right]\right\} \qquad (41)$$

$m = 1$ to ∞

where $X(m) = \dfrac{A_m}{a\gamma_m^A[B_0-C_0]}$ and $\delta\left(\frac{b}{a}m, int\right) = \begin{cases} 1 \text{ if } \frac{b}{a}m \text{ is an integer} \\ 0 \text{ if } \frac{b}{a}m \text{ is not an integer}\end{cases}$

We now write equation (33) as

$$F = \frac{a^2\epsilon_0}{\pi}\sum_{m=1}^{\infty}\frac{X(m)}{m}\sin m\pi\frac{b}{a} \qquad (42)$$

Using equations (36) and (42) we write

$$C_A = -\frac{a^2 \omega \epsilon_0}{bc\,\pi} \sum_{m=1}^{\infty} \frac{X(m)}{m} \sin m\pi \frac{b}{a} \tag{43}$$

From equations (35) and (36) we note

$$C_B = -\frac{a}{b} C_A \tag{44}$$

$$C_c = -\frac{a}{c} C_A \tag{45}$$

The remainder of this report will be concerned with determining C_A by solving equation (41) by approximate methods.

One check on the correctness of equations (41) and (43) is provided by noting in Fig. 4 that for the case $\epsilon_B = \epsilon_0$ C_A should not change when the dimensions of guides B and C are interchanged. In accordance with this physical observation it can be shown (see Appendix of reference 3) using equations (41) and (43) for the case $\mathcal{E}_B = 1$ that C_A is invariant to the transformation replacing b by c and c by b. Thus in this respect the equations check.

V. Solution of the Equations

A. Parameters

The main objective of this report is to determine the guide capacitances for various combinations of $\frac{b}{a}$, $\mathcal{E}_B$, and $\frac{2a}{\lambda_a}$. Since the analysis is based on only the T.E.M. modes propagating in each guide, there is the restriction that $\frac{2a}{\lambda_a}$, $\frac{2b}{\lambda_b}$, and $\frac{2c}{\lambda_c}$ are not greater than 1.

The values chosen for this report are

$$\frac{b}{a} = 0.1,\ 0.5,\ 0.9$$

$$\mathcal{E}_B = 1,\ 2.5,\ 10,\ 100$$

$$\frac{2a}{\lambda_a} = 0,\ 0.6,\ 1.0$$

in order that results may be obtained over a wide range of parameters.

Since it can easily be shown that $\frac{2b}{\lambda_b} = \frac{b}{a}\sqrt{\mathcal{E}_B}\,\frac{2a}{\lambda_a}$, those permutations of the parameters for which $\frac{2b}{\lambda_b} > 1$ will not be allowed.

B. Method of Approximation

The infinite number of unknowns X(m) can, in principle, be determined from the infinite set of simultaneous equations represented by equation (41). The X(m) are then used to calculate the capacitance C_A, by employing equation (43). The other capacitances immediately follow from equations (44) and (45). In practice, however, equation (41) must be solved by some method of approximation. The method utilized in this report attempts to approximate the exact answer by solving only finite sets of simultaneous linear equations.

We will solve equation (41) as a set of simultaneous equations of rank 4, 5, and 6 and from the behavior of these solutions determine an approximate capacitance. The validity of this approximation will be examined by a comparison with the exact results for the case $\mathcal{E}_B = 1$.[1]

The quantity we are interested in obtaining is the capacitance C_A given by equation (43). We define

$$C = -\frac{bc\pi}{a^2 \omega \epsilon_0} C_A = \sum_{m=1}^{\infty} \frac{X(m)}{m} \sin m\pi \frac{b}{a} \tag{46}$$

where X(m) are the solutions to the infinite set of equations represented by (41). Let the index m in equation (41) be restricted to the integers from 1 to 6. The solutions to this 6 x 6 set will be called $X_6(m)$. Thus one approximation to C could be

$$C^{(1)} = \sum_{m=1}^{6} \frac{X_6(m)}{m} \sin m\pi \frac{b}{a} \tag{47}$$

This approximation has 2 sources of error. The first is the error arising from the fact that $X_6(m) \neq X(m)$ for m = 1,6. The second is the error due to the exclusion of

$$\sum_{m=7}^{\infty} \frac{X(m)}{m} \sin m\pi \frac{b}{a}$$

It is our aim to determine a refinement of the approximation given by equation (47) based on the computed results, which will reduce the errors mentioned above.

The computer program was designed to solve the system of equations of rank 4, 5, and 6. It was felt that a plot of these results for a few typical cases and an extrapolation would indicate the probable solutions for the first four unknowns ($X(m)$, $m = 1, 4$) and also serve to exhibit the behavior of the unknowns for $m > 4$. As an example of this procedure for one set of parameters, Figs. 5, 6, and 7 show respectively the dependence of the solutions on the equation set size, a rough estimate of the values of $\Delta X(m)$ for $m = 1, 4$ extrapolated from the finite sets, and the extrapolated values of $X(m)$. An examination of the computed and graphical results reveals the following information:

(1) $X(m) \sin (m\pi\frac{b}{a})$ is always positive

(2) The extrapolated value of $X(m)$ is very close to $x_6(m)$

(3) The extrapolated value of $x(m)$ for $m > 5$ decreases roughly as $\frac{1}{m}$. Here we must make an exception for $\frac{b}{a} = 0.5$ because the results show $|x(2m + 1)| >> |x(2m)|$.

But in this case the odd values of m decrease as $\frac{1}{m}$ and these are the only values that enter into the computation of C.

Based on the above observations a suitable refinement to equation (47) is

$$C^{(2)} = C^{(1)} + 5|x_6(5)| \sum_{m=7}^{\infty} \left| \frac{\sin m\pi b/a}{m^2} \right| \tag{48}$$

The series $\sum_{m=7}^{\infty} \left| \frac{\sin m\pi b/a}{m^2} \right|$ is roughly the same if $\frac{b}{a}$ is 0.1, 0.5, or 0.9.

We note for $\frac{b}{a} = 0.5$ the value of this series can be determined from the well known result

$$\sum_{m=1}^{\infty} \left| \frac{\sin \frac{m\pi}{2}}{m^2} \right| = \sum_{m=1}^{\infty} \frac{1}{(2m-1)^2} = \frac{\pi^2}{8}$$

From this we find

$$\sum_{m=7}^{\infty} \left| \frac{\sin \frac{m\pi}{2}}{m^2} \right| = 0.0826144$$

Our approximation is now given by

$$C^{(2)} = C^{(1)} + 0.413 \left| \alpha_6(5) \right|$$

or using equation (48)

$$-\frac{a^2 C^{(2)}}{bc\pi} = \frac{C_A^{(2)}}{w\epsilon_0} = \frac{C_A^{(1)}}{w\epsilon_0} - \frac{a^2}{bc} \frac{0.413}{\pi} \left| \alpha_6(5) \right| \qquad (49)$$

where

$$\frac{C_A^{(1)}}{w\epsilon_0} = -\frac{a^2}{bc\pi} \sum_{m=1}^{6} \frac{\alpha_6(m)}{m} \sin m\pi \frac{b}{a}$$

The computed results are tabulated below. Figures 8 through 13 show these results in graphical form.

TABLE OF COMPUTED RESULTS

$\frac{b}{a}$	$\frac{2a}{\lambda_a}$ / ϵ_B	$\lvert \varkappa_6(5) \rvert$	$-\frac{C_A^{(1)}}{w\epsilon_0}$	$-\frac{C_A^{(2)}}{w\epsilon_0}$	$\lvert \varkappa_6(5) \rvert$	$-\frac{C_A^{(1)}}{w\epsilon_0}$	$-\frac{C_A^{(2)}}{w\epsilon_0}$	$\lvert \varkappa_6(5) \rvert$	$-\frac{C_A^{(1)}}{w\epsilon_0}$	$-\frac{C_A^{(2)}}{w\epsilon_0}$
		0.0			0.6			1.0		
0.1	1	.00698	.09252	.103	.00708	.1018	.112	.00822	.2441	.256
0.5	1	.01032	.2083	.214	.0112	.2451	.251	.0257	.8448	.858
0.9	1	.00698	.09252	.103	.00708	.1018	.112	.00822	.2441	.256
0.1	2.5	.0075	.09475	.106	.00762	.1042	.115	.00894	.2481	.261
0.5	2.5	.0101	.2258	.231	.01074	.2705	.276	.0285	1.385	1.40
0.9	2.5	.00712	.1112	.122	.00706	.1292	.139	-	-	-
0.1	10	.00778	.09596	.107	.00792	.1055	.117	.00935	.2504	.264
0.5	10	.00951	.2387	.244	.00977	.2916	.297	-	-	-
0.9	10	.00688	.1250	.135	-*	-	-	-	-	-
0.1	100	.00787	.09634	.108	.008	.1059	.118	.00947	.2511	.265
0.5	100	.00922	.2434	.248	-	-	-	-	-	-
0.9	100	.00672	.1302	.140	-	-	-	-	-	-

*The dash (-) indicates that $\frac{2b}{\lambda_b} = \frac{b}{a}\sqrt{\epsilon_B}\,\frac{2a}{\lambda_a} > 1$ and thus solutions with these sets of parameters are excluded from this analysis.

C. Comparison with the Results Obtained by Marcuvitz for $\epsilon_B = \epsilon_0$.

On pages 353-355 of Marcuvitz (op. cit.) the E-plane bifurcation problem is discussed and the results given graphically for an exact solution of this problem using integral transform methods. These results can be compared to those of this report for the case $\epsilon_B = 1$. We use the notation on page 353 but to avoid confusion; since b denotes different dimensions in this report and in Marcuvitz, we let the b of Marcuvitz be called b' . The dimension Marcuvitz labels b is the dimension of guide A in this report. Comparing Fig. 4 in this report to Figure 6.4 - 2 in Marcuvitz we observe

$$X = -\frac{1}{\omega C_A}$$

where $X = Z_0 \cot \frac{2\pi d}{\lambda_g}$

For this problem $\lambda_g = \lambda_a$ and the characteristic impedance is given by

$$Z_0 = \frac{b'}{w}\sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{a}{w}\sqrt{\frac{\mu_0}{\epsilon_0}}$$

Therefore

$$C_A = \frac{1}{\omega X} = -\frac{w}{\omega b' \sqrt{\frac{\mu_0}{\epsilon_0}} \cot \frac{2\pi d}{\lambda_g}}$$

Using $\omega\sqrt{\mu_0 \epsilon_0} = \frac{2\pi}{\lambda_g}$ we have

$$\frac{C_A}{w\epsilon_0} = -\frac{\tan \frac{2\pi d}{\lambda_g}}{\frac{2\pi b'}{\lambda_g}} \tag{50}$$

When $\frac{1}{\lambda_g} \to 0$ we have $$\lim_{\frac{1}{\lambda_g} \to 0} \left[\frac{C_A}{\omega\epsilon_0}\right] = -\frac{\alpha}{b'} \qquad (51)$$

Using the graph on page 354 we can construct the table below and compare $\frac{C_A}{\omega\epsilon_0}$ with the computed $\frac{C_A^{(2)}}{\omega\epsilon_0}$ of this report.

TABLE OF COMPARED RESULTS FOR $\varepsilon_b = 1$.

$\varepsilon_B = 1$

$\frac{b_1}{b'} = \frac{b}{a}$	$\frac{2b'}{\lambda_g} = \frac{2a}{\lambda_a}$	$\frac{\pi d}{b'}$	$\frac{2\pi d}{\lambda_g}$	$\tan\left(\frac{2\pi d}{\lambda_g}\right)$	$-\frac{C_A}{\omega\epsilon_0}$	$-\frac{C_A^{(2)}}{\omega\epsilon_0}$	$\left[\frac{C_A - C_A^{(2)}}{C_A}\right] 100\%$
0.1	0.0	0.32	0.0	0.0	0.102	0.103	-0.9%
0.5	0.0	0.69	0.0	0.0	0.219	0.214	2.28%
0.1	0.6	0.35	0.21	0.213	0.113	0.112	0.9%
0.5	0.6	0.76	0.456	0.49	0.260	0.251	3.46%
0.1	1.0	0.69	0.69	0.824	0.262	0.256	2.29%
0.5	1.0	1.245	1.245	2.95	0.938	0.858	8.52%

From the above results we see that in all but one case the error is within 4% of Marcuvitz' results. Thus $\frac{C_A^{(2)}}{\omega\epsilon_o}$ is a fairly good approximation to $\frac{C_A}{\omega\epsilon_o}$ for $\mathcal{E}_B = 1$. Since the structure of the equations is the same for $\mathcal{E}_B \neq 1$, and the computed results manifest no drastic change for $\mathcal{E}_B \neq 1$, we may expect that $\frac{C_A^{(2)}}{\omega\epsilon_o}$ will be within a few per cent of the correct value for all $\mathcal{E}_B$.

VI. Scattering Description

This section will be concerned with transforming the impedance description of the guide to a scattering description. Since we know now the values of the capacitances in the equivalent circuit, all other waveguide quantities such as reflection coefficient, standing wave radio, etc., may be determined. We will assume, for the sake of simplicity, that only guide B is excited and that guides A and C are terminated in their characteristic impedance. To calculate the reflection coefficient at the junction, $\Gamma(0)$, we use equation (30) given on page 14 of "Waveguide Handbook" (op. cit.).

$$\Gamma(z) = \frac{1 - Y'(z)}{1 + Y'(z)} \tag{52}$$

where $Y'(Z) = Y(Z)/Y^o$

and $Y(Z)$ is the admittance of the transmission line at z and Y^o is the characteristic admittance of the transmission line.

We note that the circuit shown in Fig. 4 corresponds to the half-space problem. However, due to symmetry, the reflection coefficient of the original guide in Fig. 1 is the same as the half-space guide. The circuit we consider is shown in Fig. 14, where the characteristic admittances are found from equation (29-a). We make the following definitions:

$$Y_A = Y_A^o + j\omega C_A \tag{53}$$

$$Y_B = j\omega C_B \tag{54}$$

$$Y_c = Y_A^o + j\omega C_c \tag{55}$$

We then have

$$Y'(0) = \frac{Y}{Y_B^o} = \frac{Y_B}{Y_B^o} + \frac{Y_A Y_c}{Y_B^o(Y_A + Y_c)} \tag{56}$$

Using equations (29-a) and (45) we write equation (55) as

$$Y_c = \frac{w}{c}\sqrt{\frac{\epsilon_0}{\mu_0}} - j\omega \frac{a}{c} C_A \tag{57}$$

We make use of the quantity $-\frac{C_A}{w\epsilon_0}$ computed in part V by defining the positive number C_V

$$C_V = -\frac{C_A}{w\epsilon_0} \tag{58}$$

We now substitute equation (58) into (57) to yield

$$Y_c = \frac{w}{c}\sqrt{\frac{\epsilon_0}{\mu_0}}\left[1 + j\omega\sqrt{\mu_0\epsilon_0}\, a\, C_V\right] \tag{59}$$

but since $\omega\sqrt{\mu_0\epsilon_0} = \frac{2\pi}{\lambda_c} = \frac{2\pi}{\lambda_a}$ equation (59) can be written as

$$Y_c = \frac{w}{c}\sqrt{\frac{\epsilon_0}{\mu_0}}\left[1 + j\left(\frac{2\pi a}{\lambda_a}\right)C_V\right] \tag{60}$$

Similarly equation (53) can be written as

$$Y_A = \frac{w}{a}\sqrt{\frac{\epsilon_0}{\mu_0}}\left[1 - j\left(\frac{2\pi a}{\lambda_a}\right)C_V\right] \tag{61}$$

After a few of these substitutions equation (56) may be written as

$$Y'(0) = \frac{A(1+C_L^2) + jC_L(A^2-1)}{\sqrt{\epsilon_B}\,(A^2+C_L^2)} \tag{62}$$

where $A = \frac{2a}{b} - 1$

and $C_L = \frac{2a\pi}{\lambda_a} C_V$

Substituting (62) into (52) we can calculate the reflection coefficient $\Gamma(0)$.

Let us now examine the behavior of the reflection coefficient for two simple cases.

<u>Case 1.</u> $\frac{2a}{\lambda_a} = 0$

If $\frac{2a}{\lambda_a} = 0$, then $C_L = 0$ and equation (62) becomes

$$Y'(0) = \frac{1}{\sqrt{\epsilon_B}\,A} \tag{63}$$

Substituting equation (63) into (52) we obtain

$$\Gamma(0) = \frac{\sqrt{\epsilon_B}\,A - 1}{\sqrt{\epsilon_B}\,A + 1} \tag{64}$$

We note when $A \rightarrow 1$

$$\Gamma(0) = \frac{\sqrt{\epsilon_B} - 1}{\sqrt{\epsilon_B} + 1}$$

and when $\sqrt{\epsilon_B} \rightarrow 1$ $\quad \Gamma(0) \rightarrow 0$

Also if $\mathcal{E}_B = 1$, equation (64) becomes

$$\Gamma(0) = 1 - \frac{b}{a} \tag{65}$$

This is in agreement with the physical result that as $\frac{b}{a} \rightarrow 1$, the guide ceases to become bifurcated and no reflections occur.

Case 2. $C_L^2 << 1$

A study of the Table of Computed Capacitance in section V-C shows that this is a fairly good approximation when $2a/\lambda_a \leq 0.6$. Since $A = \left[\frac{2a}{b} - 1\right] \geq 1$, we also have $C_L^2 << A$. With this approximation equation (62) can be written

$$Y'(0) = \frac{1}{\sqrt{\epsilon_B}\, A} + j\frac{C_L(A^2-1)}{A^2} \tag{66}$$

This tells us the obvious fact that the best match, i.e. $Y'(0) = 1$ occurs when $A \rightarrow 1$ and $\sqrt{\mathcal{E}_B} \rightarrow 1$.

It is also interesting to inquire how the wave entering from guide B is divided between guides A and C. Using equations (6-a) and (29-b) we obtain

$$V_{0A} = -A_0^r a \qquad V_{0C} = -C_0^r c \tag{67}$$

where A_0^r and C_0^r are the complex amplitudes of the outgoing waves propagating in guides A and C respectively. We have the relations

$$V_{0A} = \frac{I}{Y_A} \qquad V_{0C} = \frac{I}{Y_C} \tag{68}$$

where I is shown in Fig. 14. Combining equations (60), (61), (67), and (68) we get

$$\frac{A_0^r}{C_0^r} = \frac{1 + jC_L}{1 - jC_L} \tag{69}$$

and

$$\left|\frac{A_0^r}{C_0^r}\right| = 1 \tag{70}$$

which gives the result that the magnitudes of the transmitted waves in guides A and C are always the same. To find the ratio of the powers flowing into guides A and C, assuming them terminated by their characteristic impedance we employ the equation

$$P = \frac{1}{2}|V|^2 Y^0$$

$$\therefore \quad \frac{P_A}{P_C} = \frac{|V_{0A}|^2 Y_A^0}{|V_{0C}|^2 Y_C^0} = \frac{|A_0^r a|^2 c}{|C_0^r c|^2 a} = \frac{a}{c} \tag{71}$$

This is a general result independent of $\mathcal{E}_B$ and subject to the restriction that $\frac{2a}{\lambda} \leq 1$

We now give a table of computed $|\Gamma(0)|$ for the range of parameters considered in this problem.

Table of Computed $|\Gamma(0)|$

$\frac{b}{a}$	ϵ_B	$\frac{2a}{\lambda}$ = 0.0: $|\Gamma(0)|$	0.6: $|\Gamma(0)|$	1.0: $|\Gamma(0)|$
0.1	1	.90	.90	.90
0.5	1	.50	.50	.37
0.9	1	.10	.10	.10
0.1	2.5	.94	.93	.92
0.5	2.5	.65	.61	.35
0.9	2.5	.32	.32	-
0.1	10	.97	.98	.95
0.5	10	.81	.77	-
0.9	10	.59	-*	-
0.1	100	.99	.99	.99
0.5	100	.94	-	-
0.9	100	.85	-	-

These results show that increasing ϵ_B and decreasing $\frac{b}{a}$ and $\frac{2a}{\lambda}$ tends to increase $|\Gamma(0)|$.

*The dash (-) indicates that $\frac{2b}{\lambda_b} = \frac{b}{a}\sqrt{\epsilon_B}\,\frac{2a}{\lambda_a} > 1$ and thus solutions with these sets of parameters are excluded from this analysis.

References

1. N. Marcuvitz, "Waveguide Handbook". M.I.T. Radiation Laboratory Series, Vol. 10 (McGraw-Hill Book Company, Inc., 1951).

2. J. R. Whinnery and H. W. Jamieson, "Equivalent Circuit for Discontinuities in Transmission Lines", Proc. I.R.E., 32, 98-114, February, 1944.

3. H. M. Cronson, "Equivalent Circuit Parameters for an Inhomogeneous Bifurcated Waveguide", Master's thesis, Brown University, (1961).

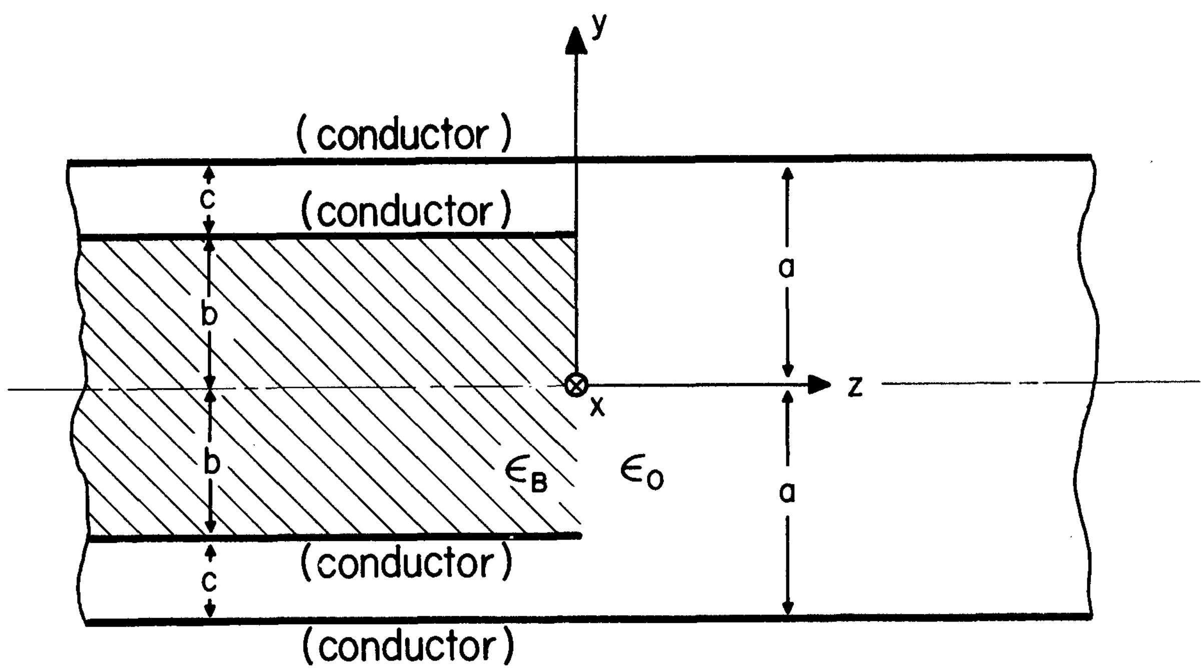


FIG. I A SEMI-INFINITE, DIELECTRIC FILLED, PARALLEL PLATE GUIDE CONTAINED SYMMETRICALLY IN AN INFINITE PARALLEL PLATE GUIDE.

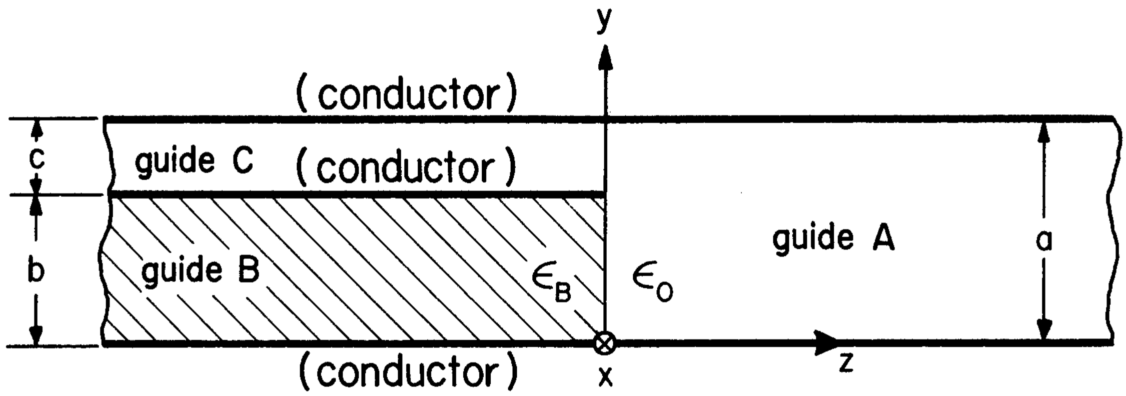


FIG. 2 THE INHOMOGENEOUS BIFURCATED GUIDE

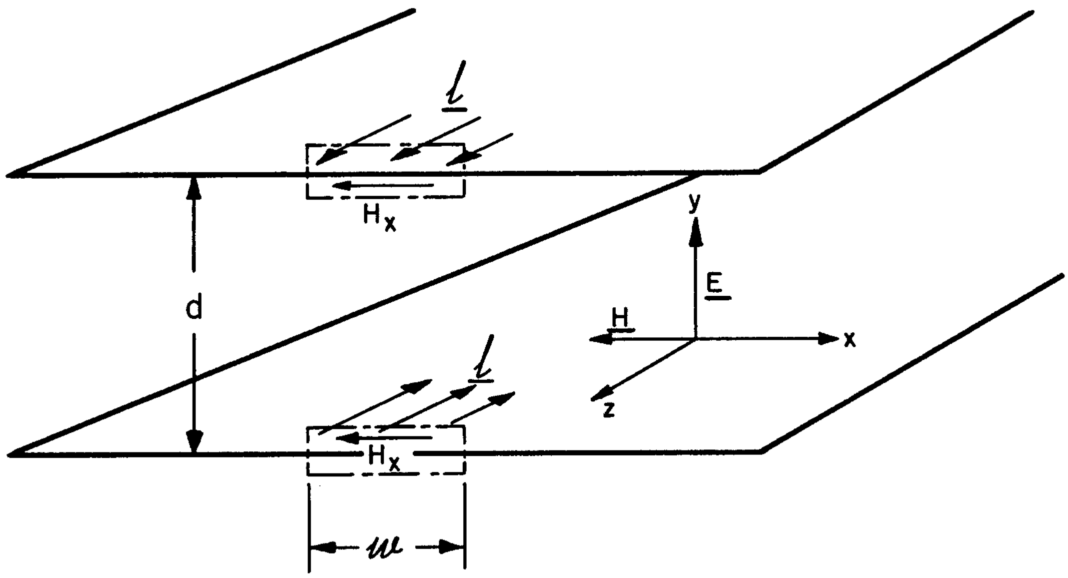


FIG. 3 ORIENTATIONS OF $\underline{l}$, w, AND H_x

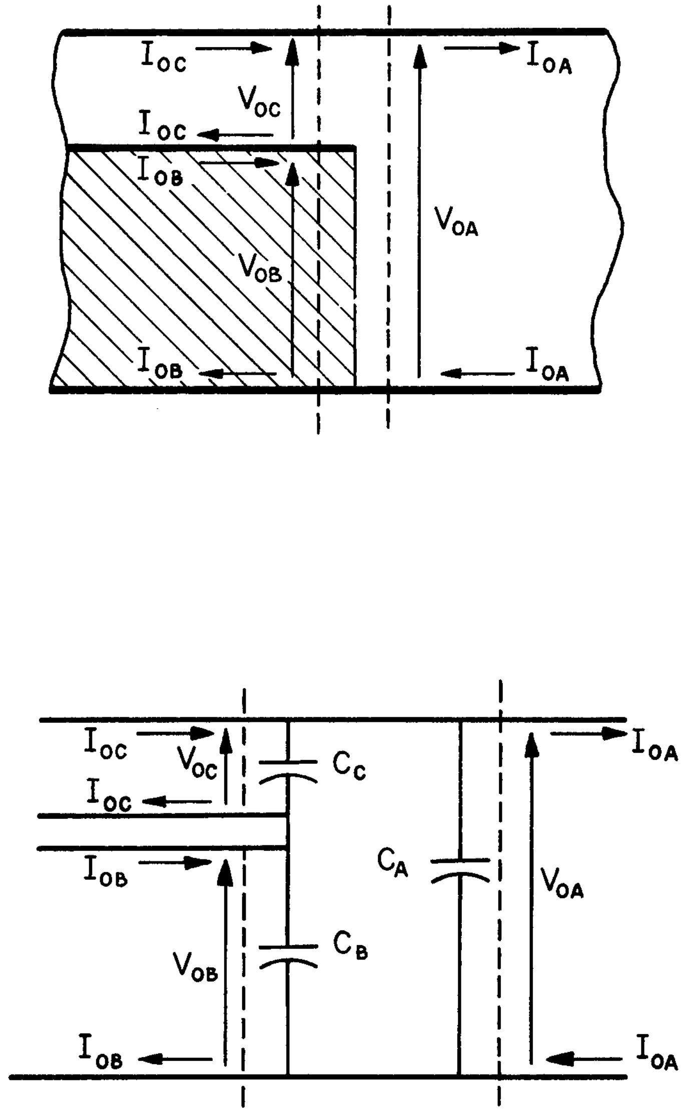


FIG. 4 EQUIVALENT CIRCUIT REPRESENTATION

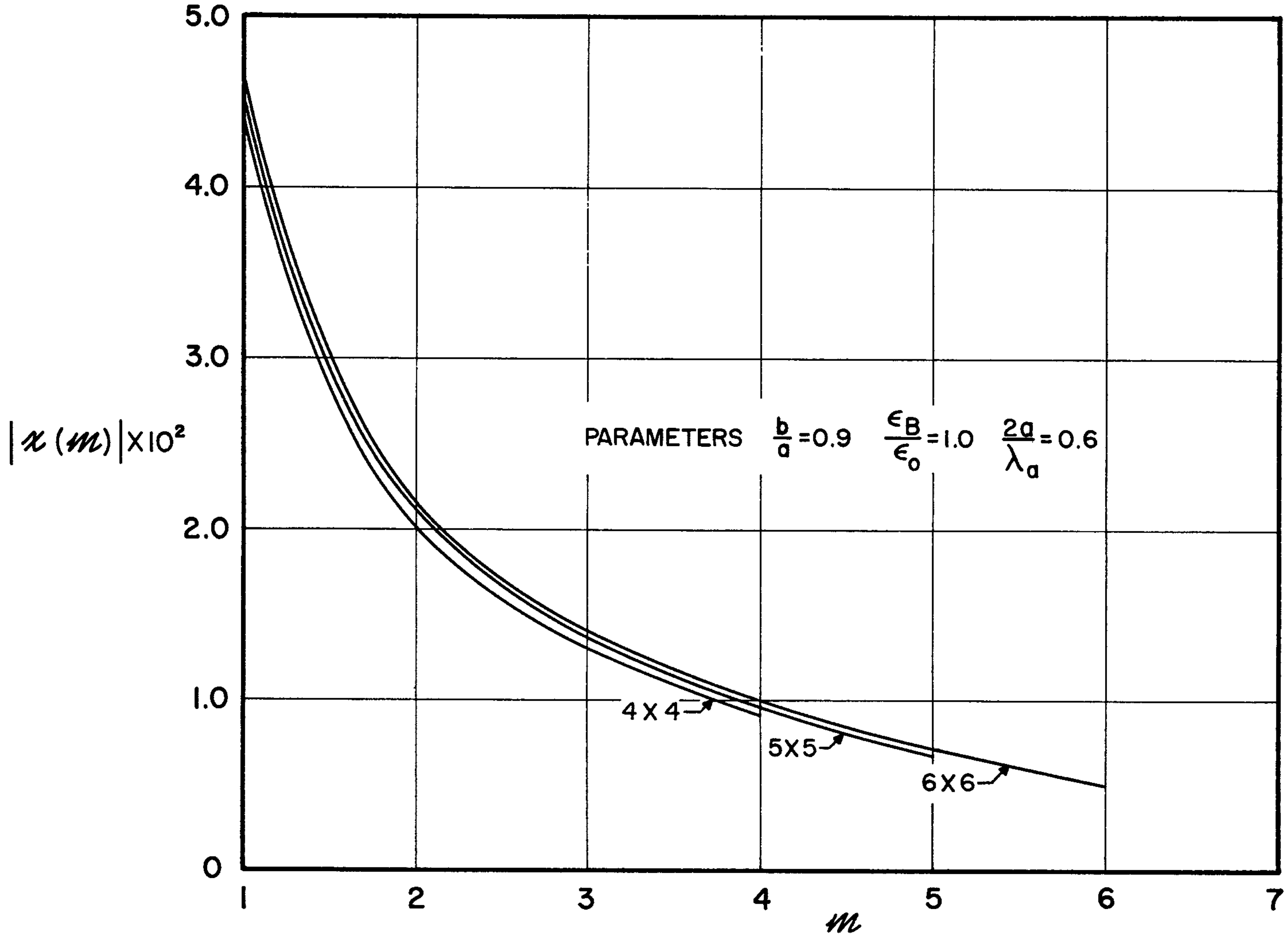


FIG. 5 THE DEPENDENCE OF THE SOLUTIONS ON THE EQUATION SET SIZE

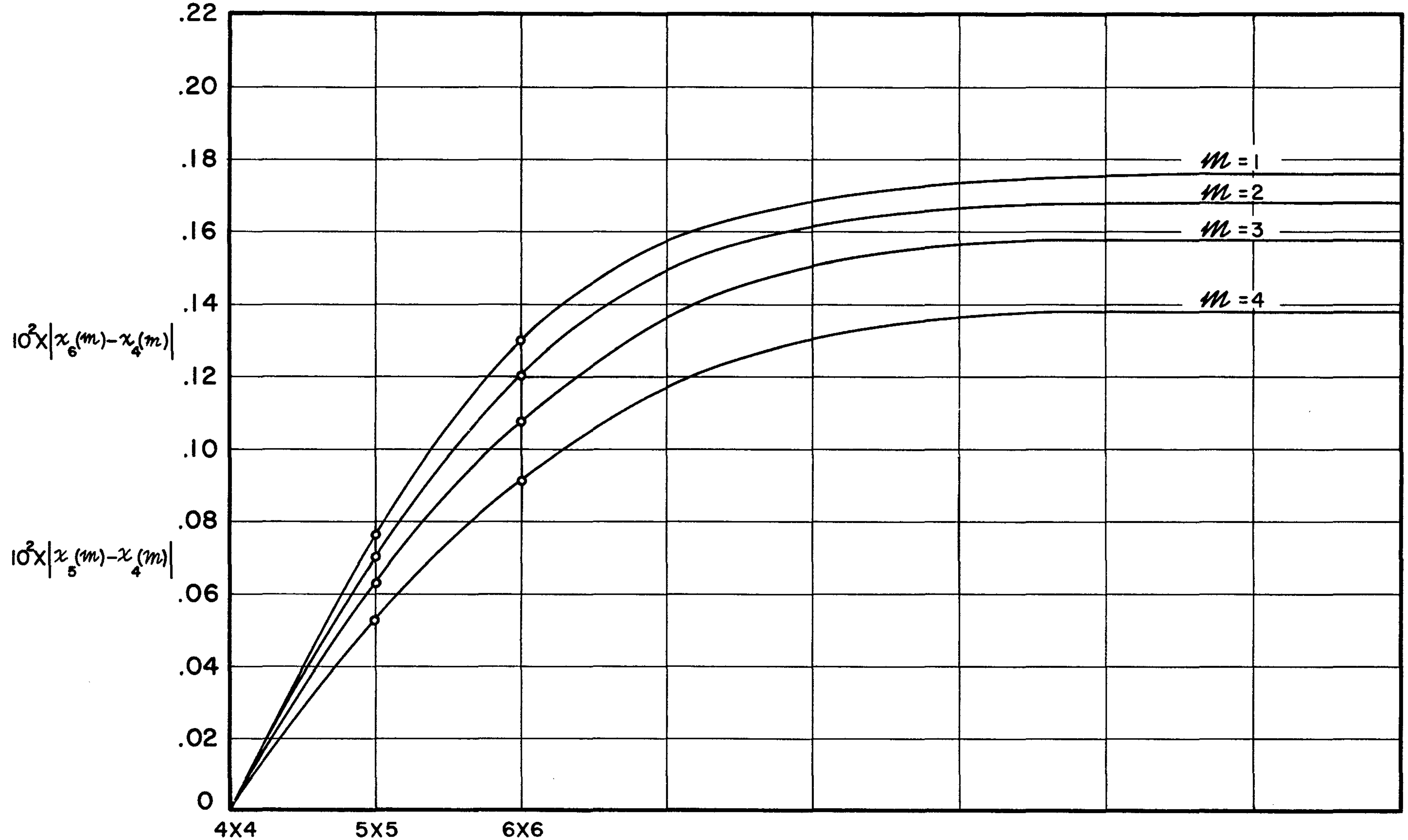


FIG. 6 EXTRAPOLATING VALUES OF $x(m)$ FOR $m = 1$ TO 4 FROM THE FINITE SETS. PARAMETERS $\frac{b}{a} = 0.9$ $\frac{\epsilon_B}{\epsilon_0} = 1.0$ $\frac{2a}{\lambda_a} = 0.6$

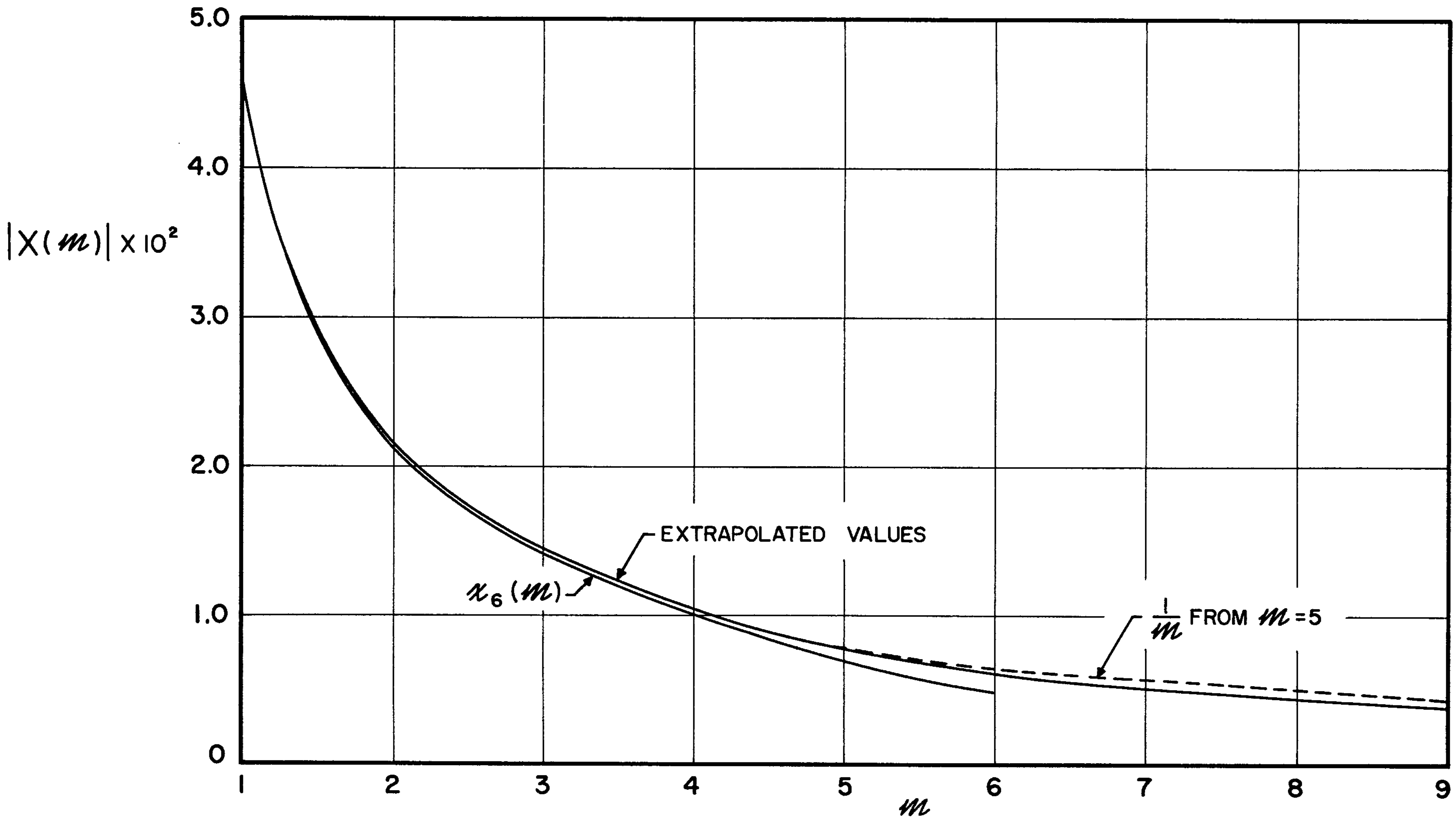
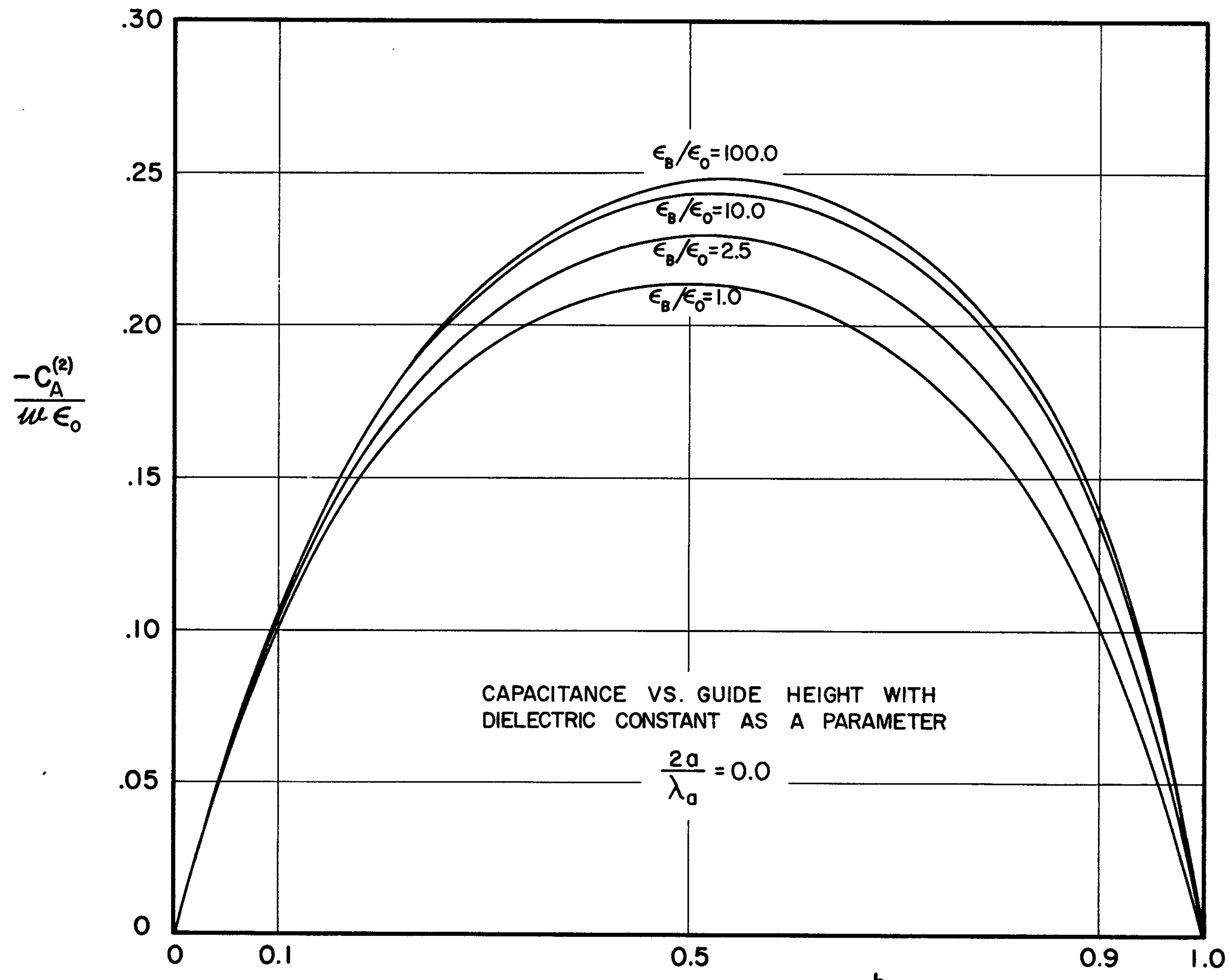


FIG. 7 EXTRAPOLATED VALUES OF $x(m)$ PARAMETERS $\frac{b}{a} = 0.9$ $\frac{\epsilon_B}{\epsilon_0} = 1.0$ $\frac{2a}{\lambda_a} = 0.6$



.30
.25
.20
.15
.10
.05
0
$\frac{-C_A^{(2)}}{w\epsilon_0}$
$\epsilon_B/\epsilon_0=100.0$
$\epsilon_B/\epsilon_0=10.0$
$\epsilon_B/\epsilon_0=2.5$
$\epsilon_B/\epsilon_0=1.0$
CAPACITANCE VS. GUIDE HEIGHT WITH
DIELECTRIC CONSTANT AS A PARAMETER
$\frac{2a}{\lambda_a}=0.0$
0
0.1
0.5
0.9
1.0
$\frac{b}{a}$

FIGURE 8

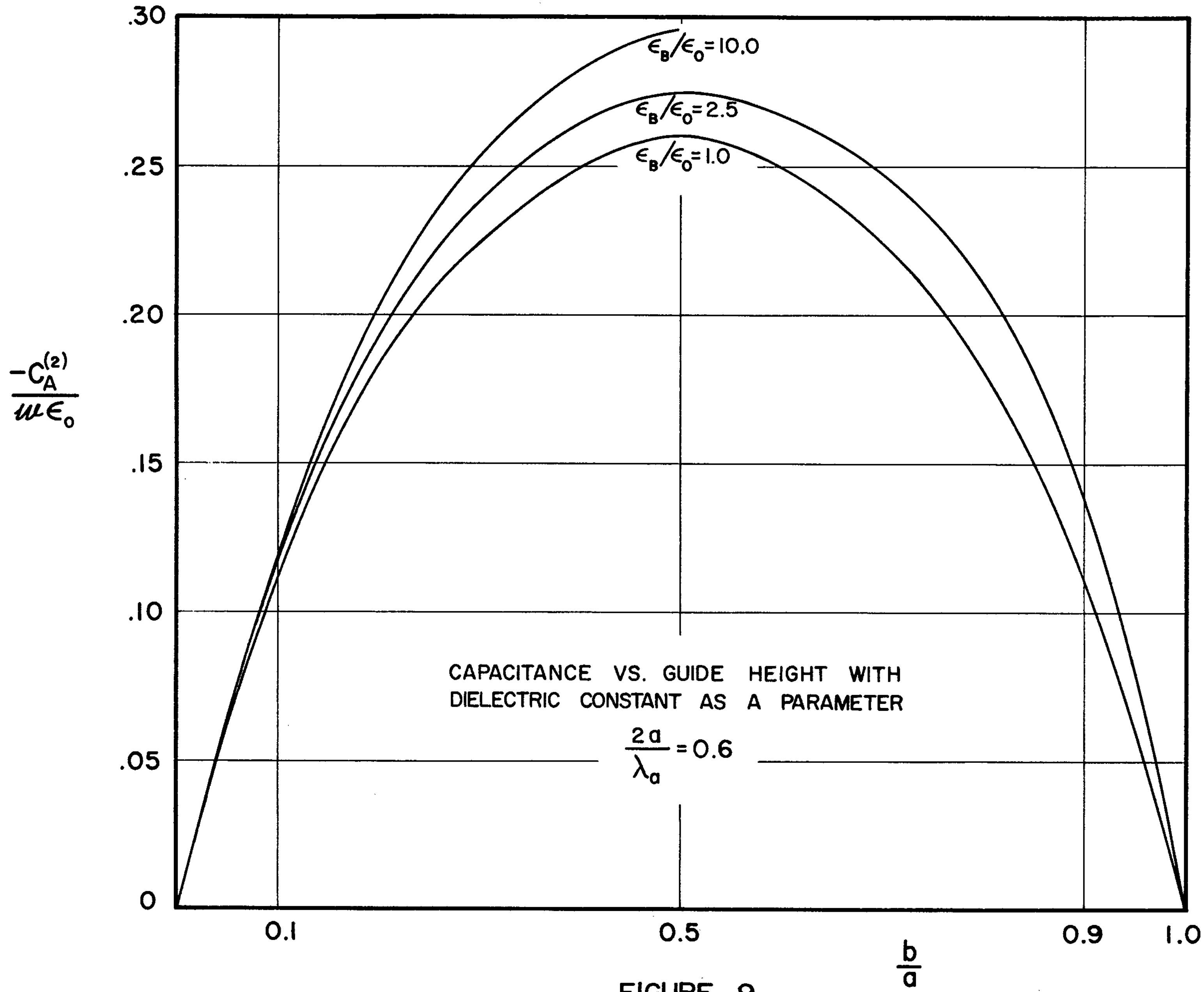
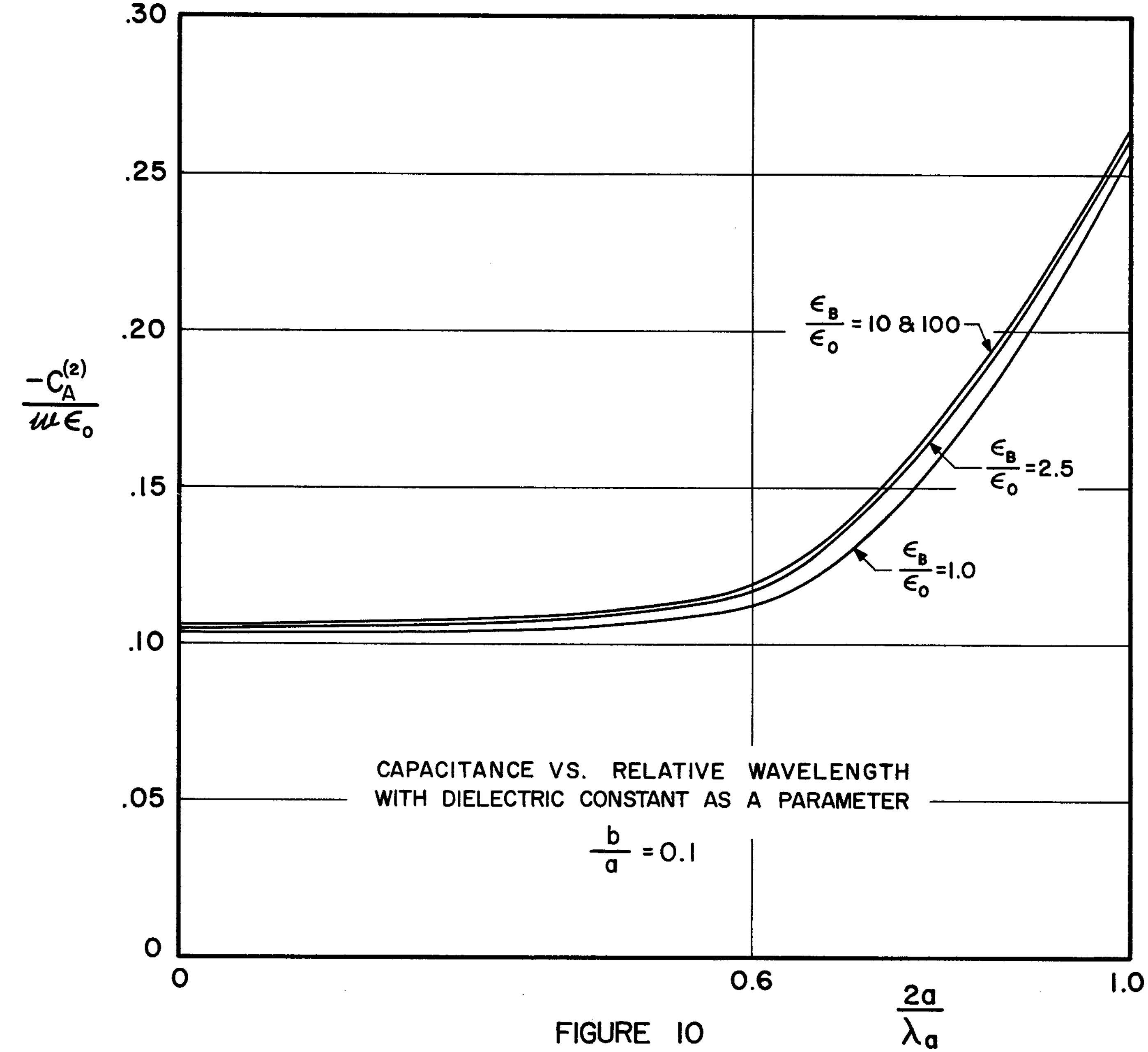
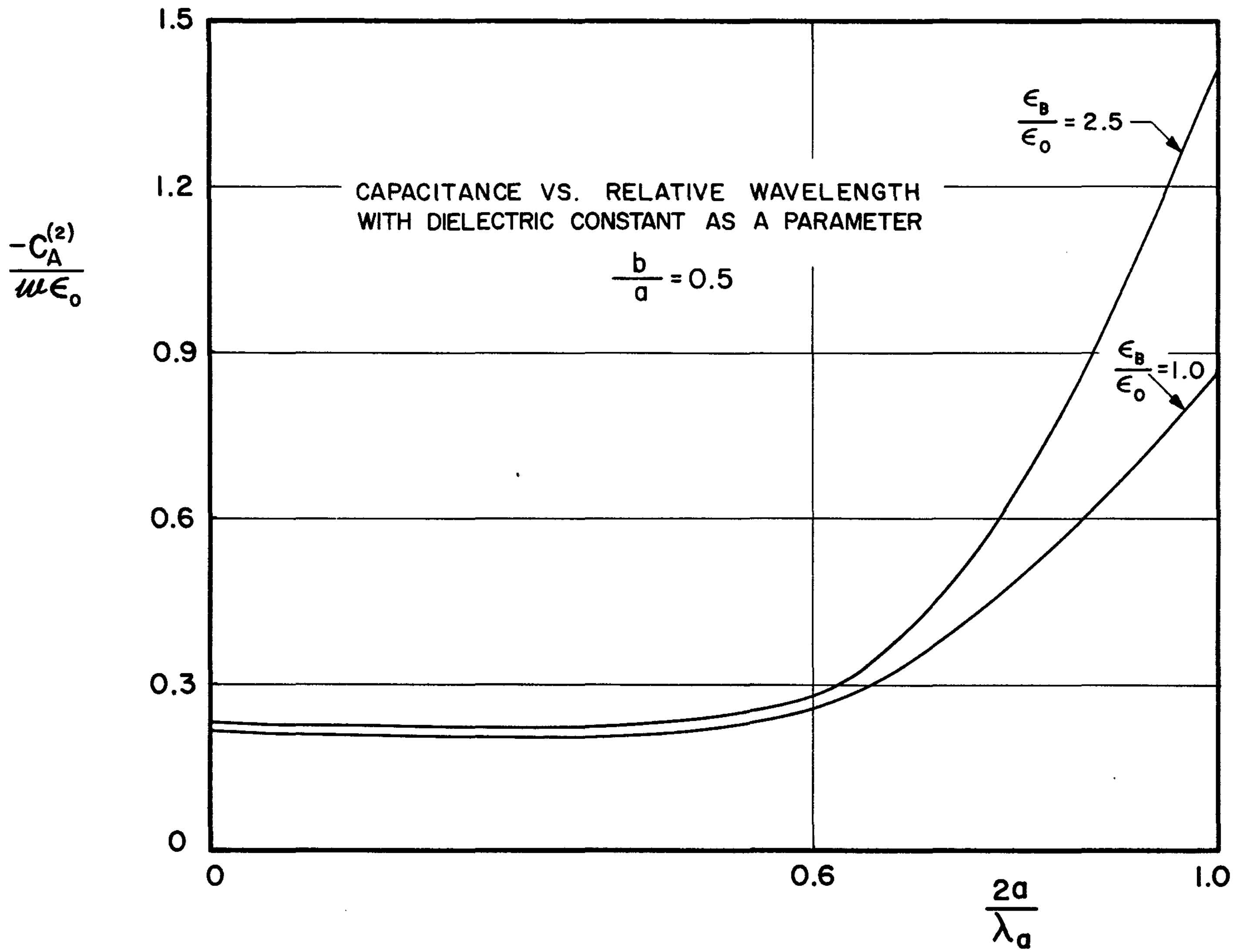

.30
.25
.20
.15
.10
.05
0
$\frac{-C_A^{(2)}}{\omega\epsilon_0}$
$\epsilon_B/\epsilon_0 = 10.0$
$\epsilon_B/\epsilon_0 = 2.5$
$\epsilon_B/\epsilon_0 = 1.0$
CAPACITANCE VS. GUIDE HEIGHT WITH
DIELECTRIC CONSTANT AS A PARAMETER
$\frac{2a}{\lambda_a} = 0.6$
0.1
0.5
0.9
1.0
$\frac{b}{a}$

FIGURE 9



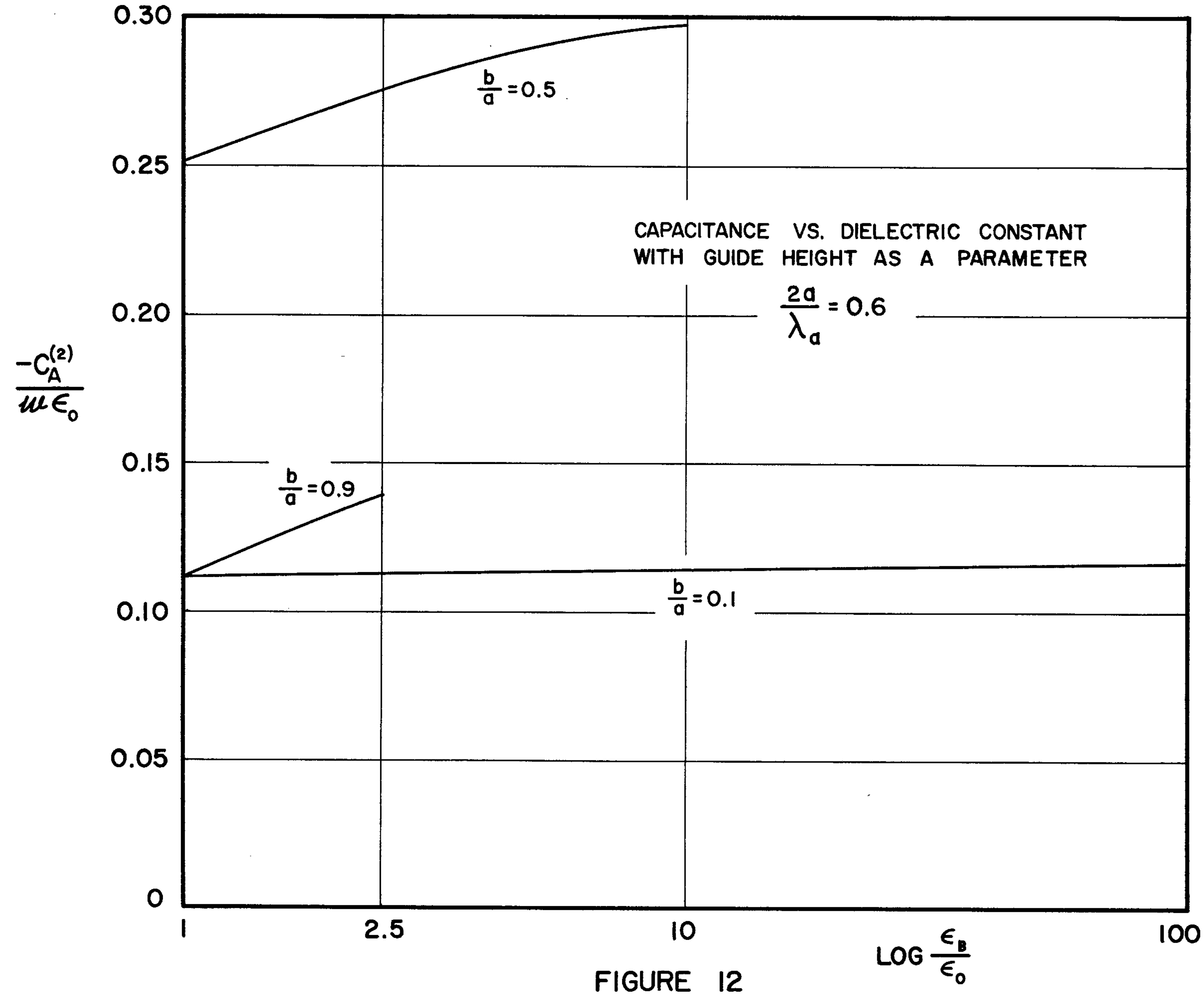
.30
.25
.20
.15
.10
.05
0
0
0.6
1.0
$\frac{-C_A^{(2)}}{\omega\epsilon_0}$
$\frac{2a}{\lambda_a}$
$\frac{\epsilon_B}{\epsilon_0} = 10 \& 100$
$\frac{\epsilon_B}{\epsilon_0} = 2.5$
$\frac{\epsilon_B}{\epsilon_0} = 1.0$
CAPACITANCE VS. RELATIVE WAVELENGTH
WITH DIELECTRIC CONSTANT AS A PARAMETER
$\frac{b}{a} = 0.1$

FIGURE 10



CAPACITANCE VS. RELATIVE WAVELENGTH
WITH DIELECTRIC CONSTANT AS A PARAMETER
$\frac{b}{a} = 0.5$
$\frac{\epsilon_B}{\epsilon_0} = 2.5$
$\frac{\epsilon_B}{\epsilon_0} = 1.0$
$\frac{-C_A^{(2)}}{\omega \epsilon_0}$
1.5
1.2
0.9
0.6
0.3
0
0.6
1.0
$\frac{2a}{\lambda_a}$

FIGURE 11



CAPACITANCE VS. DIELECTRIC CONSTANT
WITH GUIDE HEIGHT AS A PARAMETER
$\frac{2a}{\lambda_a} = 0.6$
$\frac{b}{a} = 0.5$
$\frac{b}{a} = 0.9$
$\frac{b}{a} = 0.1$
$\frac{-C_A^{(2)}}{\omega\epsilon_0}$
0.30
0.25
0.20
0.15
0.10
0.05
0
1
2.5
10
100
$\text{LOG}\ \frac{\epsilon_B}{\epsilon_0}$

FIGURE 12

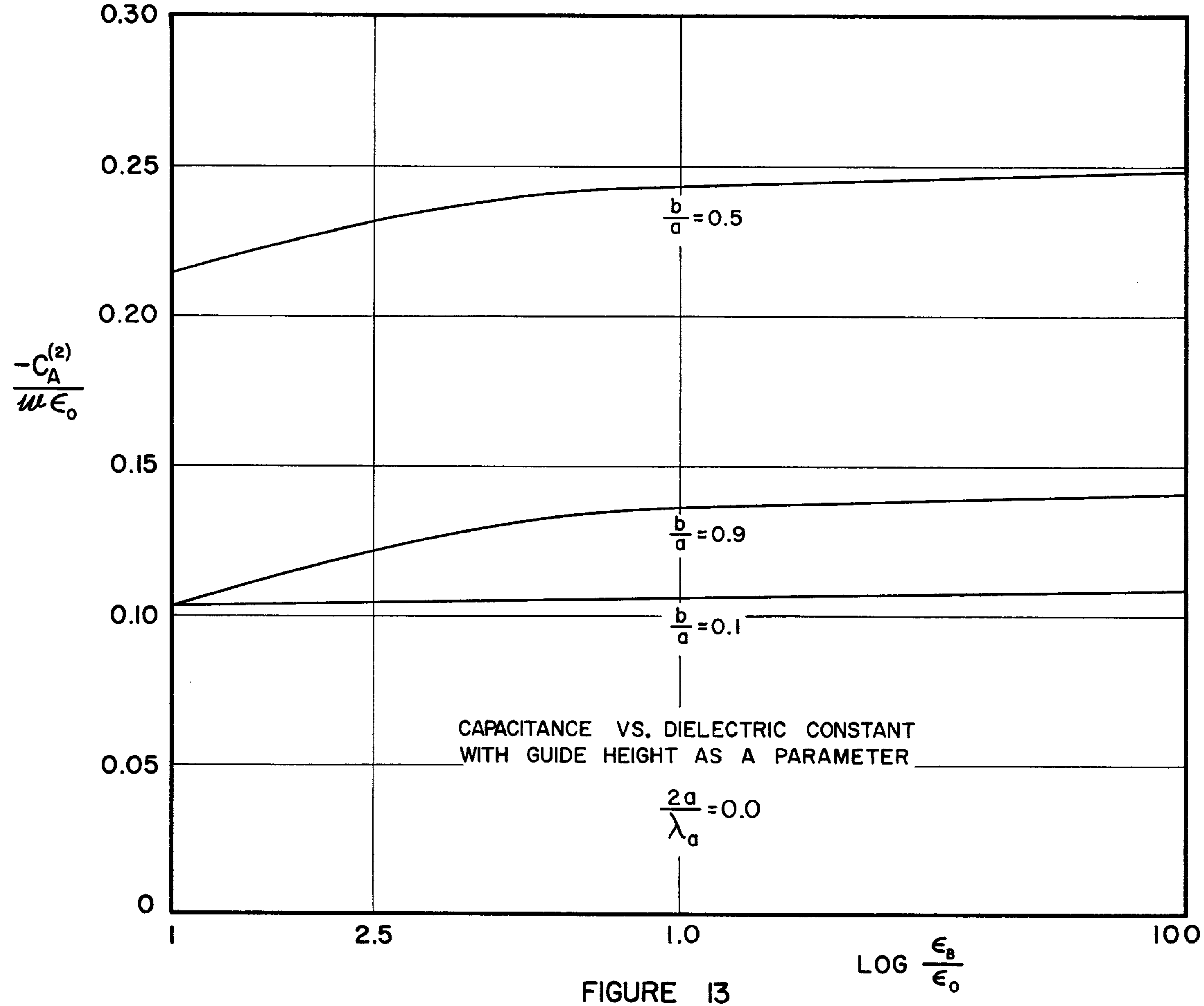

0.30
0.25
0.20
0.15
0.10
0.05
0
$\frac{-C_A^{(2)}}{\omega\epsilon_0}$
$\frac{b}{a}=0.5$
$\frac{b}{a}=0.9$
$\frac{b}{a}=0.1$
CAPACITANCE VS. DIELECTRIC CONSTANT
WITH GUIDE HEIGHT AS A PARAMETER
$\frac{2a}{\lambda_a}=0.0$
1
2.5
1.0
100
LOG $\frac{\epsilon_B}{\epsilon_0}$

FIGURE 13

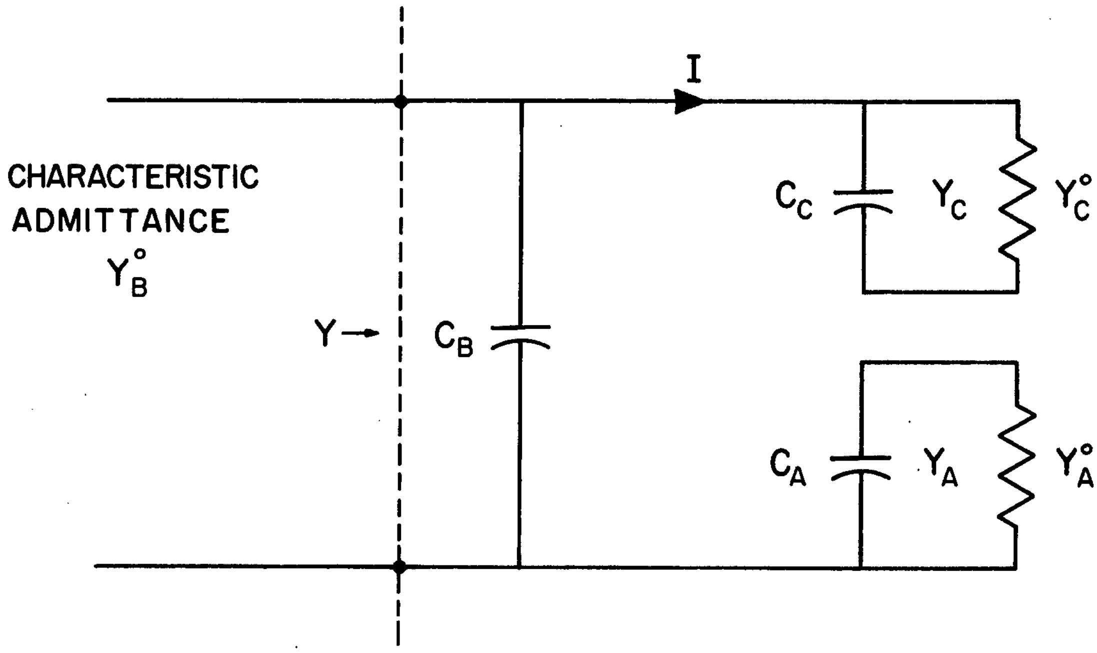


FIG. 14 EQUIVALENT CIRCUIT OF THE HALF SPACE GUIDE WITH GUIDES A AND C TERMINATED IN THEIR CHARACTERISTIC ADMITTANCES

LIST S-S

ORGANIZATION	No. of Copies
APGC (PGTRI, Tech Library) Eglin Air Force Base Florida	1
RADC (RAYLD) Attn: Documents Library Griffiss Air Force Base New York	1
AF Missile Dev. Cen. (MDGRT) Attn: Technical Library Holloman Air Force Base New Mexico	1
AUL Maxwell Air Force Base Alabama	1
AF Missile Test Center Patrick Air Force Base Florida Attn: AFMTC, Tech Library, MU-135	1
ASD (ASAPRL) Wright-Patterson Air Force Base Ohio	1
USAF Security Service (CLR) San Antonio, Texas	1
Hq. USAD (Major R. L. Stell) Tactical Air Group Washington 25, D.C.	1
AFOSR (SRY, Technical Director) 14th Street and Constitution Avenue Washington, D.C.	1
Hq. USAF (AFOAC-S/E) Communications-Electronics Directorate Washington 25, D.C.	1

ORGANIZATION	No. of Copies
Director of Air Force Research Division AFRD Washington, D.C.	1
WADD (WCLRSA, Mr. Portune) Wright-Patterson Air Force Base Ohio	1
ASD (ASRNRE-3) Attn: Mr. Paul Springer Wright-Patterson Air Force Base Ohio	1
Director, Electronics Division Air Technical Intelligence Center Attn: AFCIN-4E1, Colonel H. K. Gilbert Wright Patterson Air Force Base Ohio	1
WADD (WWDRTR, Mr. A. D. Clark) Directorate of System Engineering Dyna Soar Engineering Office Wright-Patterson Air Force Base Ohio	1
Lt. Col. Jensen (SSRTW) Space Systems Division Air Force Unit Post Office Los Angeles 45, California	1
Director U. S. Army Ordnance Ballistic Research Laboratories Aberdeen Proving Ground, Maryland Attn: Ballistic Measurements Laboratory	1
Ballistic Research Laboratories Aberdeen Proving Ground, Maryland Attn: Technical Information Branch	1
Director Evans Signal Laboratory Belmar, New Jersey Attn: Mr. O. C. Woodyard	1

ORGANIZATION	No. of Copies
Commanding General USASRDL Ft. Monmouth, New Jersey Attn: Tech. Doc. Ctr. SIGRA/SL-ADT	1
Commanding General, SIGFM/EL-PC USASRDL Forth Monmouth, New Jersey Attn: Dr. Horst H. Kedesy Deputy Chief, Chem-Physics Branch	1
Commanding General USARDL Fort Monmouth, New Jersey Attn: SIGFM/EL-AT	1
Commanding General USASRDL Fort Monmouth, New Jersey Attn: Mr. F. J. Triola	1
Commander Army Rocket & Guided Missile Agency Redstone Arsenal, Alabama Attn: Technical Library, ORDXR-OTL	1
Department of the Army Office of the Chief Signal Officer Washington 25, D.C. Attn: SIGRD-4a-2	1
Office of Chief Signal Officer Engineering and Technical Division Washington 25, D.C. Attn: SIGNET-5	1
Advisory Group on Electronic Parts Room 103, Moore School Building 200 South 33 rd Street Philadelphia 4, Pennsylvania	1

ORGANIZATION	No. of Copies
ASTIA Arlington Hall Station Arlington 12, Virginia	10
National Aeronautical Space Agency Langley Aeronautical Research Laboratory Langley, Virginia Attn: Mr. Cliff Nelson	1
Library National Bureau of Standards Boulder Laboratories Boulder, Colorado	1
National Bureau of Standards U. S. Department of Commerce Washington 25, D.C. Attn: Mr. A. G. McNish	1
National Bureau of Standards U. S. Department of Commerce Washington 25, D.C. Attn: Gustave Shapiro, Chief Engineering Electronics Section Electricity and Electronics Division	1
Director National Security Agency Fort George G. Meade, Maryland Attn: C3141 (Room 2C087)	1
AFCRL, Office of Aerospace Research (CRRELT) L. G. Hanscom Field Bedford, Massachusetts	10
AFCRL, Office of Aerospace Research (CRRD) Attn: Contract Files L. G. Hanscom Field Bedford, Massachusetts	2
AFCRL, Office of Aerospace Research (CRRD) Attn: Carlyle J. Sletten L. G. Hanscom Field Bedford, Massachusetts	3

ORGANIZATION	No. of Copies
Hq. ESD (ESRDW, Capt. John J. Hobson) L. G. Hanscom Field Bedford, Massachusetts	1
NASA (Dr. R. V. Hess) Langley Research Center Hampton, Virginia	1
Director, Avionics Division (AV) Bureau of Aeronautics Department of the Navy Washington 25, D.C.	1
Commander U. S. Naval Air Missile Test Center Point Mugu, California Attn: Code 366	1
Librarian U. S. Naval Post graduate School Monterey, California	1
Director U. S. Naval Research Laboratory Washington 25, D.C. Attn: Code 2027	1
Dr. J. I. Bohnert, Code 5210 U. S. Naval Research Laboratory Washington 25, D.C.	1
Chief of Naval Research Department of the Navy Washington 25, D.C. Attn: Code 427	1
Commanding Officer U. S. Naval Air Development Center Johnsville, Pennsylvania Attn: NADC Library	1

ORGANIZATION	No. of Copies
Commanding Officer and Director U. S. Navy Electronics Laboratory (Library) San Diego 52, California	1
Chief, Bureau of Ordnance Department of the Navy Surface Guided Missile Branch Washington 25, D.C. Attn: Code ReS1-3	1
Department of the Navy Bureau of Aeronautics Technical Data Division, Code 4106 Washington 25, D.C.	1
Commander U. S. Naval Air Test Center Patuxent River, Maryland Attn: ET-315, Antenna Branch	1
ARDC Regional Office c/o Department of the Navy Room 4549, Munitions Building Washington 25, D.C.	1
Airborne Instruments Laboratory, Inc. Division of Cutler Hammer Walt Whitman Road Milville, L. I. New York Attn: Library	1
Aircom, Inc. 48 Cummington Street Boston, Massachusetts	1
ACF Industries, Inc. Southeast Corner, 52nd Avenue and Jackson Street Bladensburg, Prince George's County Maryland Attn: Librarian	1

ORGANIZATION	No. of Copies
Battelle Memorial Institute 505 King Avenue Columbus 1, Ohio Attn: Wayne E. Rife, Project Leader Electrical Engineering Division	1
Bell Aircraft Corporation Post Office Box One Buffalo 5, New York Attn: Eunice P. Hazelton, Librarian	1
Bell Telephone Laboratories Murray Hill New Jersey	1
Bell Telephone Laboratories, Inc. Whippany Laboratory Whippany, New Jersey Attn: Technical Information Library	1
Aerospace Corp. Box 95085 Los Angeles 45, California Attn: Library	1
Bendix Radio Division Bendix Aviation Corporation E. Joppa Road Towson 4, Maryland Attn: Dr. D. M. Allison, Jr. Director Engineering & Research	1
Bjorksten Research Laboratories, Inc. P. O. Box 265 Madison, Wisconsin Attn: Librarian	1
Boeing Airplane Company Pilotless Aircraft Division P. O. Box 3707 Seattle 24, Washington Attn: R. R. Barber, Library Supervisor	1

ORGANIZATION	No. of Copies
Chance Vought Aircraft, Inc. 9314 West Jefferson Street Dallas, Texas Attn: A. D. Pattullo, Librarian	1
Chance Vought Corporation Vought Electronics Division P. O. Box 5907 Dallas 22, Texas	1
Chu Associates P. O. Box 387 Whitcomb Avenue Littleton, Massachusetts	1
Collins Radio Company 855 35th Street, N.E. Cedar Rapids, Iowa Attn: Dr. R. L. McCreary	1
Collins Radio Company 1200 North Alma Road Richardson, Texas Attn: C. D. Tipton	1
Convair, A Division of General Dynamics Corp. Fort Worth, Texas Attn: K. G. Brown Division Research Librarian	1
Convair, A Division of General Dynamics Corp. 3165 Pacific Highway San Diego 12, California Attn: Mrs. Dora B. Burke Engineering Librarian	1
Cornell Aeronautical Laboratory, Inc. 4455 Genesee Street Buffalo 21, New York Attn: Librarian	1

ORGANIZATION	No. of Copies
Dalmo Victor Company A Division of Textron, Inc. 1515 Industrial Way Belmont, California Attn: Mary Ellen Addems, Technical Librarian	1
Dorne and Margolin, Inc. 29 New York Avenue Westbury, Long Island, New York	1
Douglas Aircraft Company, Inc. 827 Lapham Street El Segundo, California Attn: Engineering Library	1
Douglas Aircraft Company, Inc. 3000 Ocean Park Boulevard Santa Monica, California Attn: Peter Duyan, Jr. Chief, Electrical/Electronics Section	1
Douglas Aircraft Company, Inc. 2000 North Memorial Drive Tulsa, Oklahoma Attn: Engineering Librarian, D-250	1
Electromagnetic Research Corporation 5001 College Avenue College Park, Maryland Attn: Mr. Martin Katzin	1
Electronics Communication 1830 York Road Timonium, Maryland	1
Electronic Specialty Company 5121 San Fernando Road Los Angeles 39, California Attn: Donald L. Margerum Chief Engineer, Radiating Systems Division	1
Emerson Electric Mfg. Company 8100 West Florissant Avenue St. Louis 21, Missouri Attn: Mr. E. R. Breslin, Librarian	1

ORGANIZATION	No. of Copies
Emerson Radio-Phonograph Corp. Emerson Research Laboratories 701 Lamont Street, N.W. Washington 10, D.C. Attn: Mrs. R. Corbin, Librarian	1
Fairchild Aircraft-Missiles Division Fairchild Eng. and Airplane Cor. Hagerstown, Maryland Attn: Library	1
Federal Telecommunication Laboratories 500 Washington Avenue Nutley 10, New Jersey Attn: Technical Library	1
General Electric Company Electronics Park Syracuse, New York Attn: Documents Library B. Fletcher, Building 3-143A	1
General Electric Company Missile and Space Vehicle Department 3198 Chestnut Street Philadelphia, Pennsylvania Attn: Documents Library	1
Gabriel Electronics Division Main and Pleasant Streets Millis, Massachusetts Attn: Dr. Edward Altshuler	1
General Electric Comapny 3750 D Street Philadelphia 24, Pennsylvania Attn: Mr. H. G. Lew Missile and Space Vehicle Department	1

ORGANIZATION	No. of Copies
General Precision Laboratory, Inc. 63 Bedford Road Pleasantville, New York Attn: Librarian	1
Goodyear Aircraft Corporation 1210 Massillon Road Akron 15, Ohio Attn: Library, Plant G	1
Granger Associates Electronic Systems 974 Commercial Street Palo Alto, California Attn: John V. N. Granger, President	1
The Hallicrafters Company 5th and Kostner Avenues Chicago 24, Illinois Attn: Henry Hodara, Head of Space Communication	1
Hughes Aircraft Company Antenna Department Building 12, Mail Station 2714 Culver City, California Attn: Dr. W. H. Kummer	1
Hughes Aircraft Company Florence Avenue and Teale Streets Culver City, California Attn: Louis L. Bailin Manager, Antenna Dept.	1
ITT Laboratories 3700 East Pontiac Street Fort Wayne 1, Indiana Attn: Technical Library	1
Jansky and Bailey, Inc. 1339 Wicsonsin Avenue, N.W. Washington 7, D.C. Attn: Mr. Delmer C. Ports	1

ORGANIZATION	No. of Copies
Dr. Henry Jasik, Consulting Engineer 298 Shames Drive Brush Hollow Industrial Park Westbury, New York	1
Lockheed Aircraft Corporation 2555 N. Hollywood Way California Division Engineering Library Department 72-25, Plant A-1, Building 63-1 Burbank, California Attn: N. C. Harnois	1
Lockheed Aircraft Corporation Missile Systems Division Research Library Box 504, Sunnyvale, California Attn: Miss Eva Lou Robertson, Chief Librarian	1
The Martin Company P. O. Box 179 Denver 1, Colorado Attn: Mr. Jack McCormick	1
The Martin Company Baltimore 3, Maryland Attn: Engineering Library Antenna Design Group	1
McDonnell Aircraft Corporation, Dept. 644 Box 516, St. Louis 66, Missouri Attn: C. E. Zoller Engineering Library	1
Melpar, Inc. 3000 Arlington Boulevard Falls Church, Virginia Attn: Engineering Technical Library	1
Hughes Aircraft Company Attn: Mr. L. Stark, Microwave Dept. Radar Laboratory, P. O. Box 2097 Building 600, Mail Station C-152 Fullerton, California	1

ORGANIZATION	No. of Copies
The Mitre Corporation 244 Wood Street Lexington 73, Massachusetts Attn: Mrs. Jean E. Claflin, Librarian	1
Motorola, Inc. Phoenix Research Laboratory 3102 N. 56th Street Phoenix, Arizona Attn: Dr. A. L. Aden	1
National Research Council Radio & Electrical Engineering Division Ottawa, Ontario, Canada Attn: Dr. G. A. Miller, Head Microwave Section	1
North American Aviation, Inc. 12214 Lakewood Boulevard Downey, California Attn: Technical Information Center (495-12) Space & Information Systems Division	1
North American Aviation, Inc. Los Angeles International Airport Los Angeles 45, California Attn: Engineering Technical File	1
Northrop Corporation Norair Division 1001 E. Broadway Hawthorne, California Attn: Mr. E. A. Freitas, Library Dept. 3145	1
Page Communications Engineers, Inc. 2001 Wisconsin Avenue, N.W. Washington 7, D.C. Attn: (Mrs.) Ruth Temple, Librarian	1
Philco Corporation Research Division Union Meeting Pond Blue Bell, Pennsylvania Attn: Research Librarian	1

ORGANIZATION	No. of Copies
Pickard and Burns, Inc. 240 Highland Avenue Needham 94, Massachusetts Attn: Dr. Richard H. Woodward	1
Polytechnic Research & Development Co., Inc. 202 Tillary Street Brooklyn 1, New York Attn: Technical Library	1
Radiation Engineering Laboratory Main Street Maynard, Massachusetts Attn: Dr. John Ruze	1
Radiation, Inc. Melbourne, Florida Attn: R F Systems Division Technical Information Center	1
Radiation Systems, Inc. 440 Swann Avenue Alexandria, Virginia Attn: Library	1
RCA Laboratories David Sarnoff Research Center Princeton, New Jersey Attn: Miss Fern Cloak, Librarian	1
Radio Corporation of America Defense Electronic Products Building 10, Floor 7 Camden 2, New Jersey Attn: Mr. Harold J. Schrader, Staff Engineer Organization of Chief Technical Administrator	1
Radio Corporation of America Surface Communications Systems Laboratory 75 Varick Street New York 13, New York	1

ORGANIZATION	No. of Copies
Radio Corporation of America West Coast Missile and Surface Radar Division Engineering Library Building 306/2 Attn: L. R. Hund, Librarian 8500 Balboa Boulevard Van Nuys, California	1
Radio Corporation of America Defense Electronic Products Advanced Military Systems Princeton, New Jersey Attn: Mr. David Shore	1
Director, USAF Project RAND Via: AF Liaison Office The Rand Corporation 1700 Main Street Santa Monica, California	1
The Rand Corporation 1700 Main Street Santa Monica, California Attn: Technical Library	1
Rantec Corporation 23999 Ventura Boulevard Calabasas, California Attn: Grace Keener, Office Manager	1
Raytheon Company Wayland Laboratory Wayland, Massachusetts Attn: Miss Alice G. Anderson, Librarian	1
Raytheon Company Missile Systems Division Hartwell Road Bedford, Massachusetts Attn: Donald H. Archer	1

ORGANIZATION	No. of Copies
Republic Aviation Corporation Farmingdale, Long Island, New York Attn: Engineering Library	1
Sanders Associates, Inc. 95 Canal Street Nashua, New Hampshire Attn: Mr. Norman R. Wild	1
Sandia Corporation Sandia Base P. O. Box 5800 Alburque, New Mexico Attn: Classified Document Division	1
Sandia Corporation Attn: Organization 1423 Sandia Base Alburque, New Mexico	1
Scanwell Laboratories, Inc. 6601 Scanwell Lane Springfield, Virginia	1
STL Technical Library Document Acquisitions Space Technology Laboratories, Inc. P. O. Box 95001 Los Angeles 45, California	1
Sperry Gyroscope Company Great Neck, Long Island, New York Attn: Florence W. Turnbull Engineering Librarian	1
Stanford Research Institute Documents Center Menlo Park, California Attn: Acquisitions	1

ORGANIZATION	No. of Copies
Sylvania Electric Products, Inc. 100 First Avenue Waltham 54, Massachusetts Attn: Charles A. Thornhill, Report Librarian Waltham Laboratories Library	1
Sylvania Reconnaissance Systems Lab. Box 188 Mountain View, California Attn: Marvin D. Waldman	1
TRG, Inc. 400 Border Street East Boston, Massachusetts Attn: Dr. Alan F. Kay	1
A. S. Thomas, Inc. 355 Providence Highway Westwood, Massachusetts Attn: A. S. Thomas, President	1
Texas Instruments, Inc. 6000 Lemmon Avenue Dallas 9, Texas Attn: John B. Travis Systems Planning Branch	1
Westinghouse Electric Corp. Electronics Division Friendship Int'l Airport Box 1897 Baltimore 3, Maryland	1
Brown University Department of Electrical Engineering Providence, Rhode Island Attn: Dr. C. M. Angulo	1
California Institute of Technology Jet Propulsion Laboratory 4800 Oak Grove Drive Pasadena, California Attn: Mr. I. E. Newlan	1

ORGANIZATION	No. of Copies
California Institute of Technology 1201 E. California Street Pasadena, California Attn: Dr. C. Papas	1
Space Sciences Laboratory Leuschner Observatory University of California Berkeley 4, California Attn: Dr. Samuel Silver, Professor of Engineering Science and Director, Space Sciences Laboratory	1
University of California Electronics Research Lab. 332 Cory Hall Berkeley 4, California Attn: J. R. Whinnery	1
University of Southern California University Park Los Angeles, California Attn: Dr. Raymond L. Chuan Director, Engineering Center	1
Case Institute of Technology Electrical Engineering Department 10900 Euclid Avenue Cleveland, Ohio Attn: Professor Robert Plonsey	1
Columbia University Department of Electrical Engineering Morningside Heights New York, New York Attn: Dr. Schlesinger	1
University of Southern California University Park Los Angeles 7, California Attn: Z. A. Kaprielian Associate Professor of Electrical Engineering	1

ORGANIZATION	No. of Copies
Cornell University School of Electrical Engineering Ithaca, New York Attn: Professor G. C. Dalman	1
University of Florida Department of Electrical Engineering Gainesville, Florida Attn: Professor M. H. Latour, Library	1
Library Georgia Technology Research Institute Engineering Experiment Station 722 Cherry Street, N.W. Atlanta, Georgia Attn: Mrs. J. H. Crosland, Librarian	1
Harvard University Technical Reports Collection Gordon McKay Library 303 Pierce Hall Oxford Street Cambridge 38, Massachusetts Attn: Librarian	1
University of Illinois Documents Division Library Urbana, Illinois	1
University of Illinois College of Engineering Urbana, Illinois Attn: Dr. P. E. Mayes, Department of Electrical Engineering	1
Illinois Institute of Technology 3301 S. Dearborn Street Chicago 16, Illinois Attn: Dr. George I. Cohn	1

ORGANIZATION	No. of Copies
Illinois Institute of Technology Technology Center Department of Electrical Engineering Chicago 16, Illinois Attn: Paul C. Yuen Electronics Research Laboratory	1
The John Hopkins University Homewood Campus Baltimore 18, Maryland Attn: Dr. Donald E. Kerr, Department of Physics	1
The Johns Hopkins University Applied Physics Laboratory 8621 Georgia Avenue Silver Spring, Maryland Attn: Mr. George L. Seielstad	1
University of Kansas Electrical Engineering Department Lawrence, Kansas Attn: Dr. H. Unz	1
Lowell Technological Institute Research Foundation P. O. Box 709 Lowell, Massachusetts Attn: Dr. Charles R. Mingins	1
Massachusetts Institute of Technology Research Laboratory of Electronics Building 26, Room 327 Cambridge 39, Massachusetts Attn: John H. Hewitt	1
Massachusetts Institute of Technology Lincoln Laboratory P. O. Box 73 Lexington 73, Massachusetts Attn: Mary A. Granese, Librarian	1

ORGANIZATION	No. of Copies
McGill University Montreal, Canada Attn: Professor G. A. Woonton Director, The Eaton Electronics Research Laboratory	1
University of Michigan Electronic Defense Group Engineering Research Institute Ann Arbor, Michigan Attn: J. A. Boyd, Supervisor	1
University of Michigan Engineering Research Institute Willow Run Laboratories Willow Run Airport Ypsilanti, Michigan Attn: Librarian	1
University of Minnesota Minneapolis 14, Minnesota Attn: Mr. Robert H. Stumm, Library	1
Physical Science Laboratory New Mexico College of Agriculture and Mechanics Arts State College, New Mexico Attn: Mr. H. W. Haas	1
New York University Institute of Mathematical Sciences Room 802 25 Waverly Place New York 3, New York Attn: Professor Morris Kline	1
Northwestern University Microwave Laboratories Evanston, Illinois Attn: R. E. Beam	1

ORGANIZATION	No. of Copies
Ohio State University Research Foundation 1314 Kinnear Road Columbus 8, Ohio Attn: Dr. T. E. Tice Department of Electrical Engineering	1
The University of Oklahoma Research Institute Norman, Oklahoma Attn: Professor C. L. Farrar, Chairman Electrical Engineering	1
The Pennsylvania State University Department of Electrical Engineering University Park, Pennsylvania	1
University of Pennsylvania Institute of Cooperative Research 3400 Walnut Street Philadelphia, Pennsylvania Attn: Department of Electrical Engineering	1
Polytechnic Institute of Brooklyn Microwave Research Institute 55 Johnson Street Brooklyn, New York Attn: Dr. Arthur A. Oliner	1
Polytechnic Institute of Brooklyn Microwave Research Institute 55 Johnson Street Brooklyn, New York Attn: Mr. A. E. Laemmel	1
Purude University Department of Electrical Engineering Lafayette, Indiana Attn: Dr. Schultz	1
Library W. W. Hansen Laboratory of Physics Stanford University Stanford, California	1

ORGANIZATION	No. of Copies
Syracuse University Research Institute Collendale Campus Syracuse 10, New York Attn: Dr. C. S. Grove, Jr. Director of Engineering Research	1
Technical University Oestervoldgade 10 G Copenhagen, Denmark Attn: Professor Hans Lottrup Knudsen	1
University of Tennessee Ferris Hall W. Cumberland Avenue Knoxville 16, Tennessee	1
The University of Texas Electrical Engineering Research Lab. P. O. Box 8026 University Station Austin 12, Texas Attn: Mr. John R. Gerhardt Assistant Director	1
The University of Texas Defense Research Laboratory Austin, Texas Attn: Claude W. Horton, Physics Library	1
University of Toronto Department of Electrical Engineering Toronto, Canada Attn: Professor G. Sinclair	1
University of Washington Department of Electrical Engineering Seattle 5, Washington Attn: D. K. Reynolds	1
University of Wisconsin Department of Electrical Engineering Madison, Wisconsin Attn: Dr. Scheibe	1